Application of CDCC to many-body breakup reaction

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Introduction

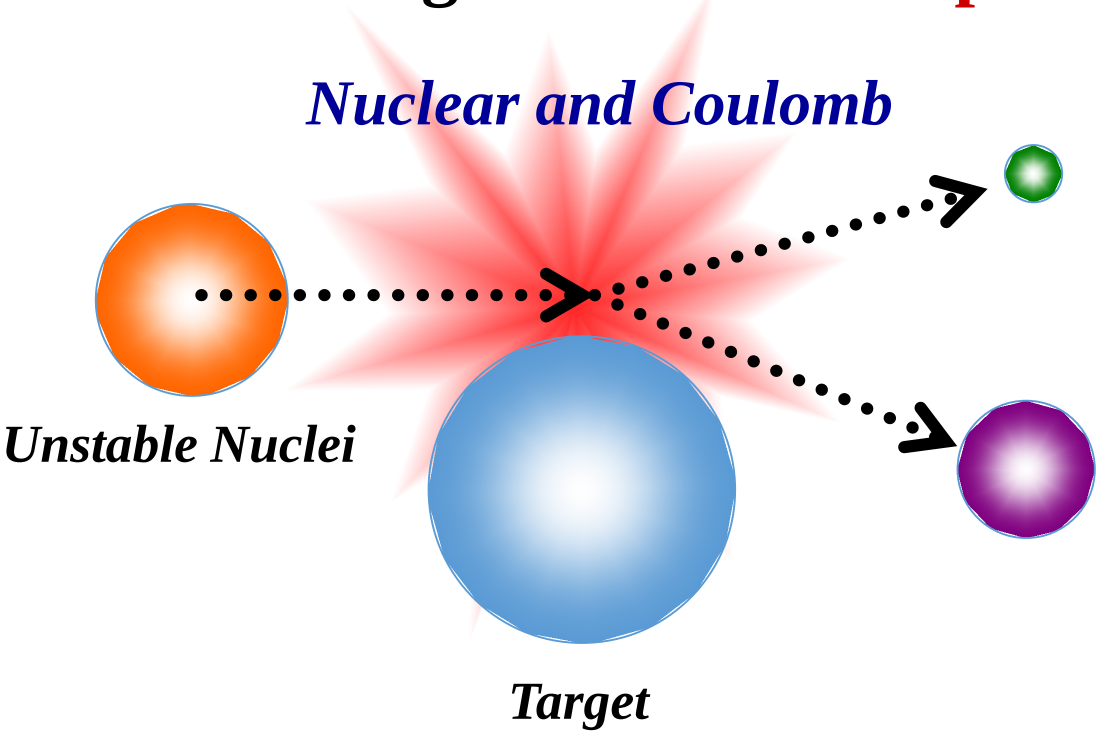
- **CDCC: Continuum-Discretized Coupled-Channels method**
  - **Developed by Kyushu group about 30 years ago**

    M. Kamimura, et al., Prog. Theor. Phys. Suppl. 89, 1 (1986)
  - **Proposed as a method of treating three-body reaction system.**
  - **Successful for analyses of nuclear and Coulomb breakup reactions**
  - **Extended to describe many-body reaction system**
  - **Applied to study of unstable nuclei, cosmology and astrophysics, and nuclear engineering.**

Yahiro, Ogata, TM, Minomo, Prog. Theor. Exp. Phys. 2012, 01A206.

Study of Unstable nuclei

- The unstable nuclear structure can be efficiently investigated via **breakup reactions**.



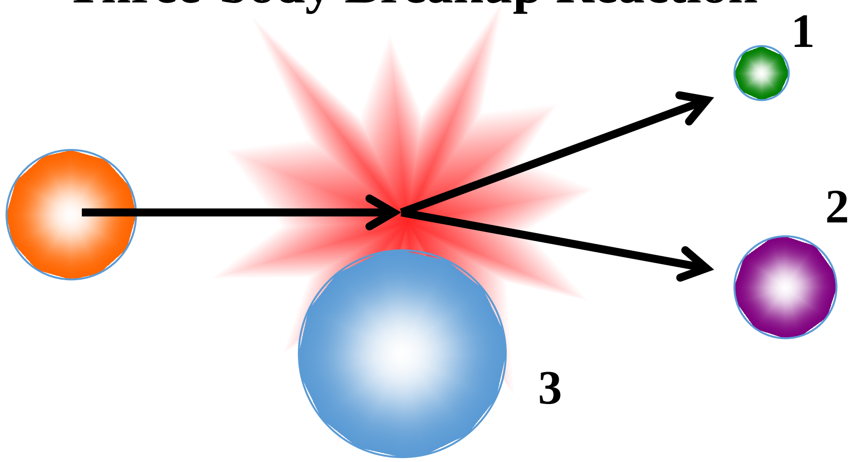
- Inelastic cross section
- Breakup cross section
- Momentum distribution of emitted particles

Structure information

- **An accurate method** of treating breakup processes is highly desirable.

Three-Body and Four-Body Breakup

Three-body Breakup Reaction



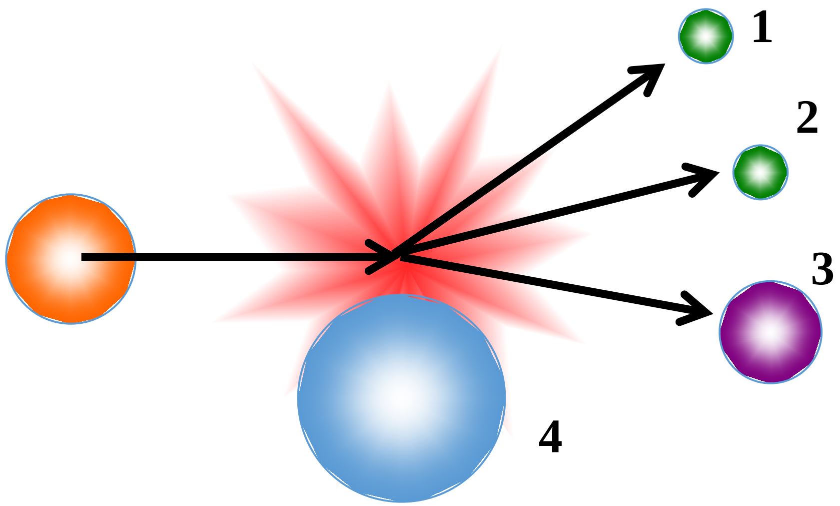
The projectile breaks up into 2 particles.

Projectile (2-body) + target (1-body)
→ 3-body breakup reaction

Ex.) d, ^{6}Li, ^{11}Be, ^{8}B, ^{15}C, etc..

One-neutron halo

Four-body Breakup Reaction



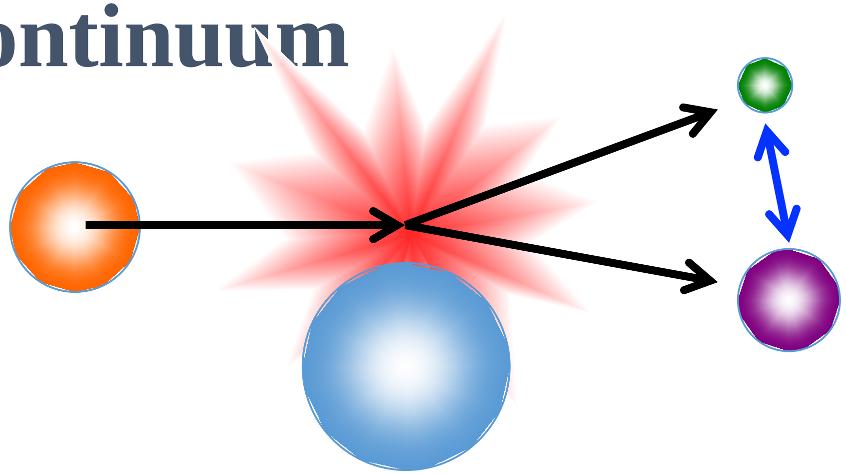
The projectile breaks up into 3 particles.

Projectile (3-body) + target (1-body)
→ 4-body breakup reaction

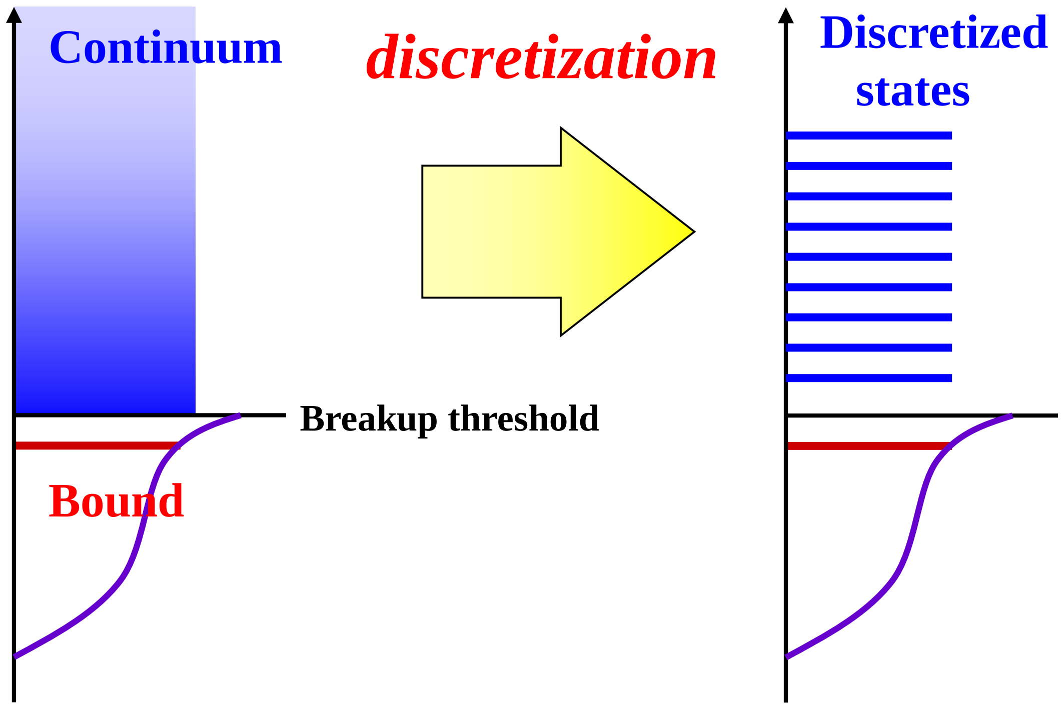
Ex.) ^{6}He, ^{11}Li, ^{14}Be, etc..

Two-neutron halo

Discretization of Continuum



☐ Essence of CDCC



- Breakup continuum states of the projectile are described by a finite number of **discretized states**

- A set of eigenstates forms **a complete set within a finite model space** that is important for breakup processes

How to describe discretized continuum states

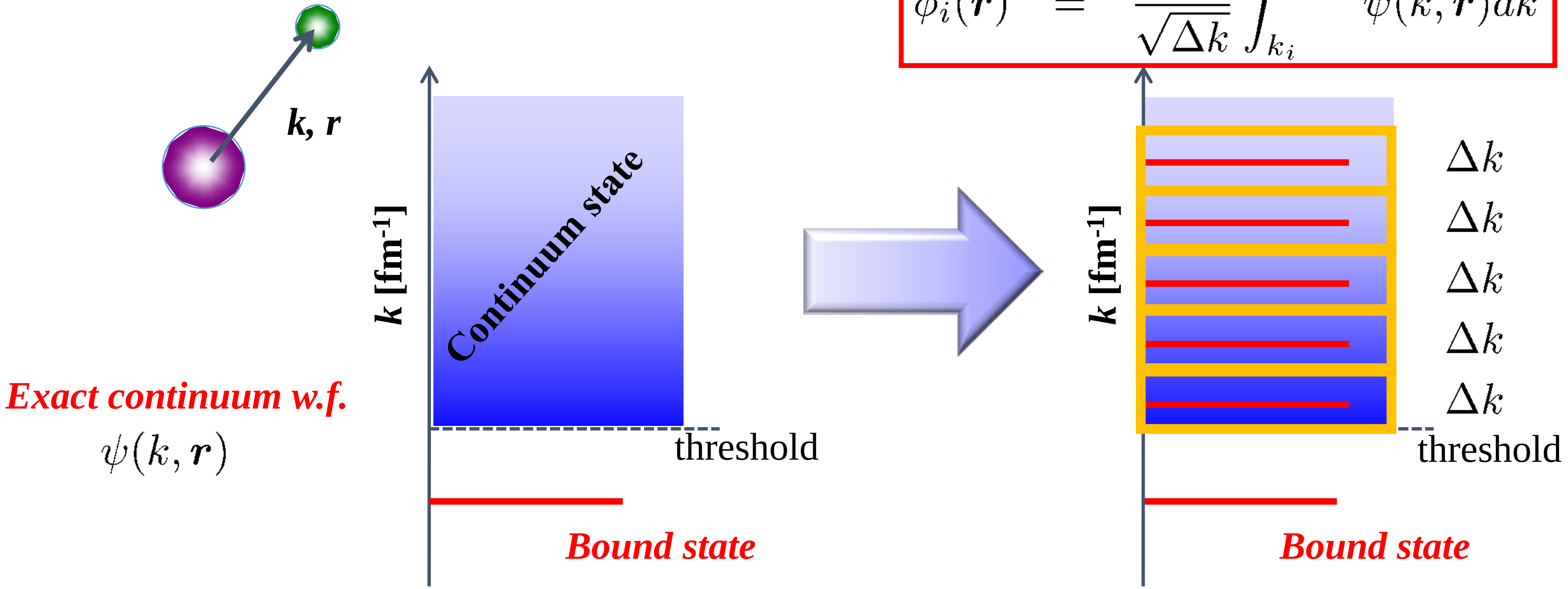
Momentum-bin Method

Average method (Momentum-bin)

➢ G. H. Rawitscher, Phys. Rev. C 9 (1974), 2210

Two-body system

$$\hat{\phi}_i(\boldsymbol{r}) = \frac{1}{\sqrt{\Delta k}} \int_{k_i}^{k_{i+1}} \psi(k, \boldsymbol{r}) dk$$



Three-body system

Application of the average method is not easy for four-body breakup reactions, because it requires *the exact three-body continuum states*.

M. M. Rodriguez-Gallardo, *et al*. Phys. Rev. C 80, 051601(R) (2009).

Pseudostate Method

Pseudo-state method

- E. W. Schmid and H. Ziegelmann, *The Quantum Mechanical Three-Body Problem* p. 192, 1974.
- R. C. Johnson and P. C. Tandy, Nucl. Phys. A 235 (1974), 56.
- B. Anders and A. Lindner, Nucl. Phys. A 296 (1978), 77.

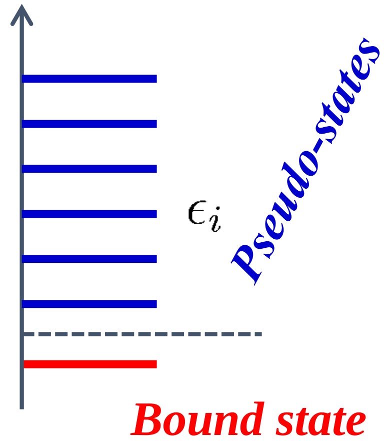
Wave function of discretized state

$$\hat{\phi}_i(\boldsymbol{r}) = \sum_{n=1}^{N} C_n^{(i)} \varphi_n(\boldsymbol{r})$$

$\varphi_n(\boldsymbol{r})$: Basis function (Gaussian, HH, …)

$C_n^{(i)}$ can be obtained by diagonalizing H of the projectile

$$\left[\left(H_{nn'} \right) - \epsilon_i \left(N_{nn'} \right)\right]\left(C_n^{(i)} \right) = 0$$



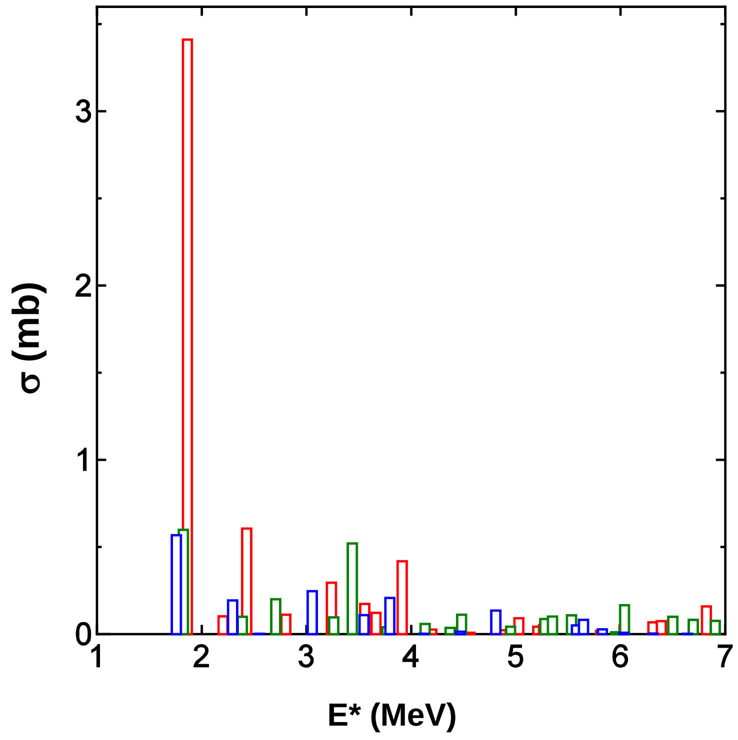
The pseudo-state method can be easily applicable to many-body breakup reactions.

Breakup Cross Section

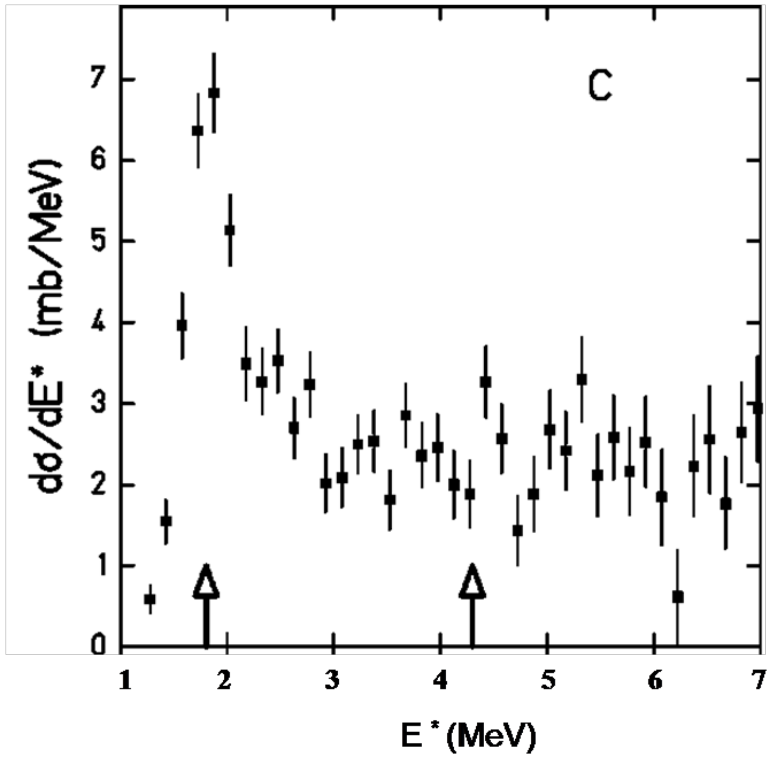
Breakup cross sections calculated by CDCC are discrete in the internal energy of the projectile.

$^{6}He+^{12}C$ scattering at 240 MeV/nucl.

4-body CDCC calc.



PRC59, 1252(1999), T. Aumann *et al*.



How to calculate the continuuous breakup cross section

Smoothing Function Method

Continuous breakup T-matrix element

$$T(k,P) = \langle \phi(k)\chi(P)|U|\Psi^{\mathrm{CDCC}}\rangle$$

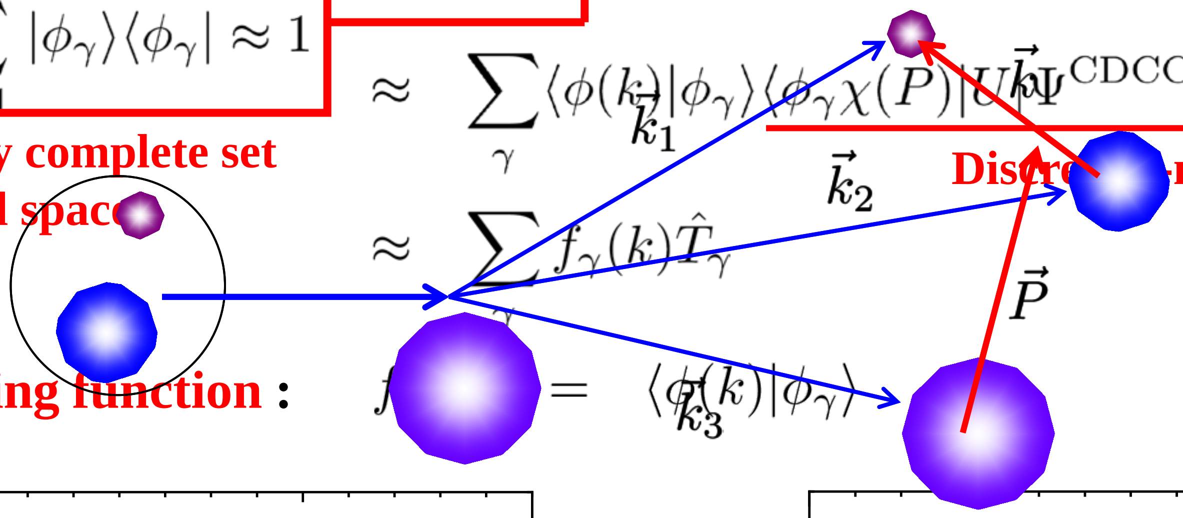
$$\approx \sum_{\gamma} \langle \phi(k)|\phi_\gamma\rangle\langle\phi_\gamma\chi(P)|U|\Psi^{\mathrm{CDCC}}\rangle$$

$$\approx \sum_{\gamma} f_\gamma(k)\hat{T}_\gamma$$

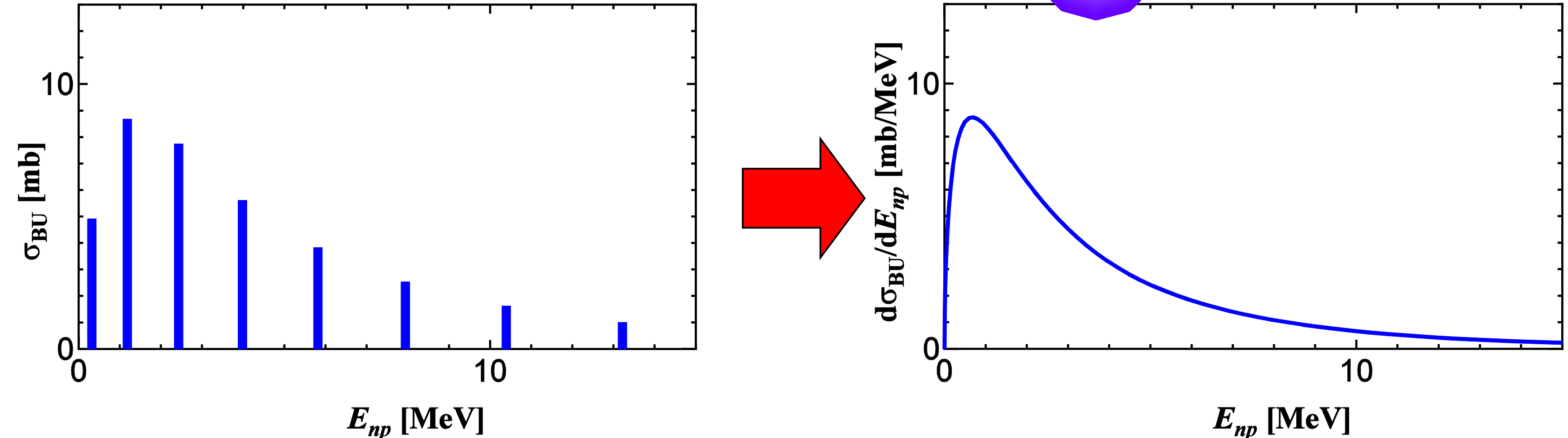
$$|\phi_0\rangle\langle\phi_0| + \sum_{\gamma=1}^{\gamma_{\max}} |\phi_\gamma\rangle\langle\phi_\gamma| \approx 1$$

Approximately complete set within a model space

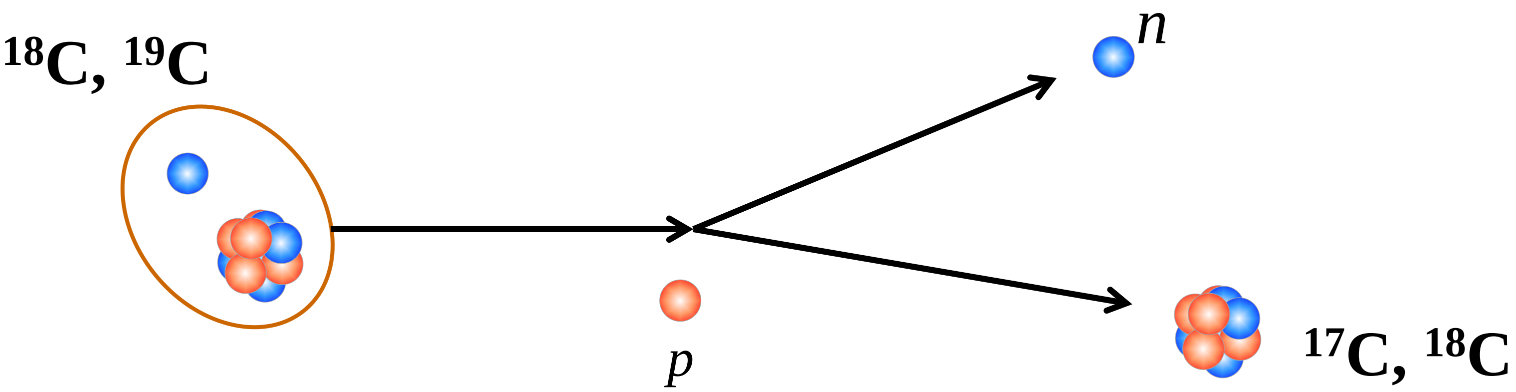
Discrete T-matrix element



Smoothing function : $f_\gamma(k) = \langle \phi(k)|\phi_\gamma\rangle$



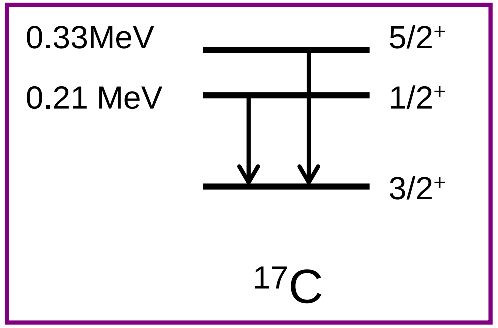
Neutron removal cross section

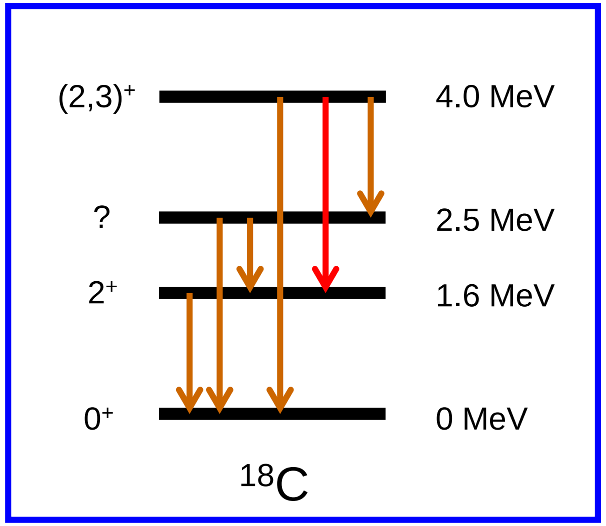


Momentum distribution

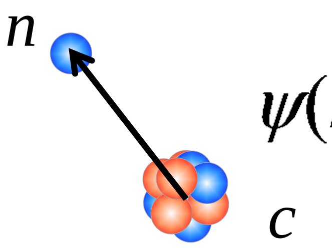
$$\psi_{\rm g.s.}(^{18}{\rm C}:0^+) = \alpha|0d_{3/2}\otimes^{17}{\rm C}(3/2^+)\rangle + \beta|1s_{1/2}\otimes^{17}{\rm C}(1/2^+)\rangle + \gamma|0d_{5/2}\otimes^{17}{\rm C}(5/2^+)\rangle$$

$$\psi_{\rm g.s.}(^{19}{\rm C}:1/2^+) = \alpha|1s_{1/2}\otimes^{18}{\rm C}(0^+)\rangle + \beta|0d_{3/2}\otimes^{18}{\rm C}(2^+)\rangle + \gamma|0d_{5/2}\otimes^{18}{\rm C}(2^+)\rangle + \zeta|0d_{5/2}\otimes^{18}{\rm C}(3^+)\rangle$$





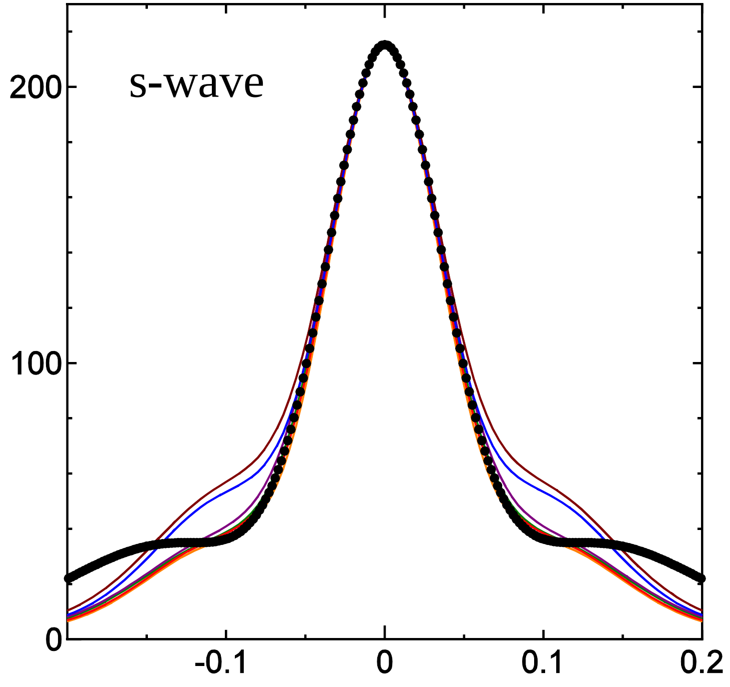
Momentum distribution

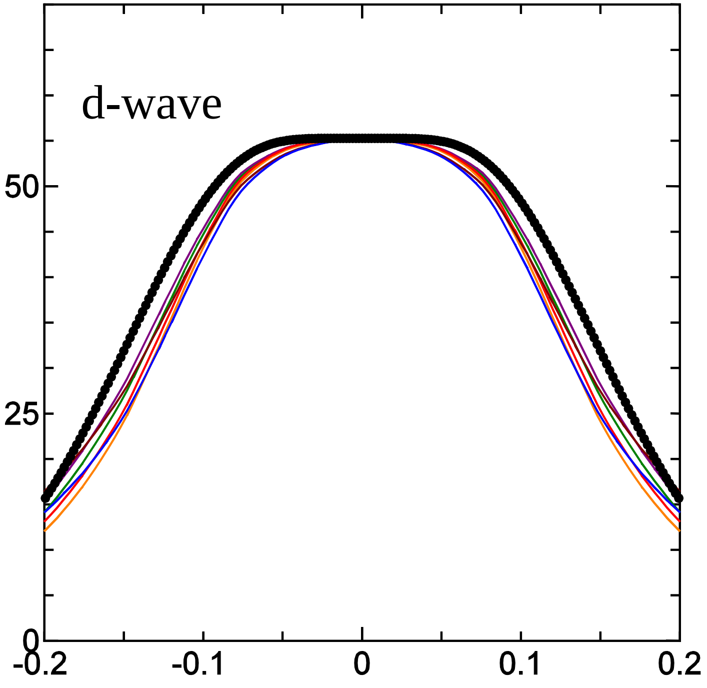


$\psi(k,r)$

Black circle : Fourier transformation of the w.f.

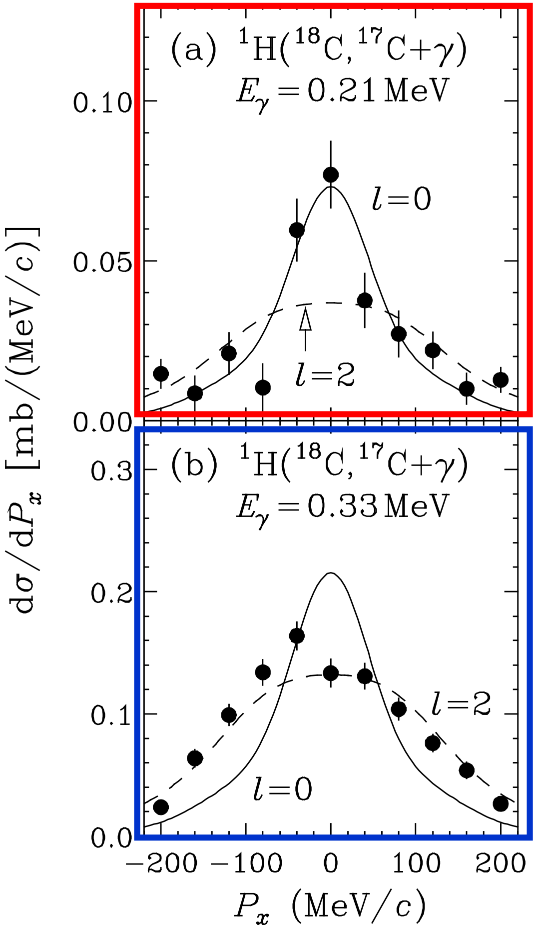
$$f(k_z) = \int e^{ik_z z} \psi(k,r)\, dr$$





Momentum distribution corresponds to the Fourier transformation of the wave function.

Analyses of ^{17}C and ^{18}C

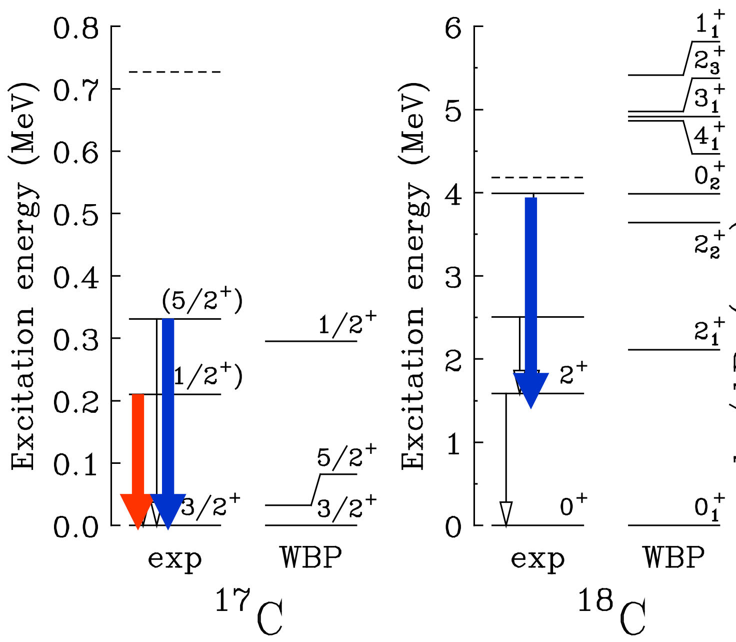


Neutron in d-wave

$^{17}C(E_x=0.33MeV)$:$5/2^+$

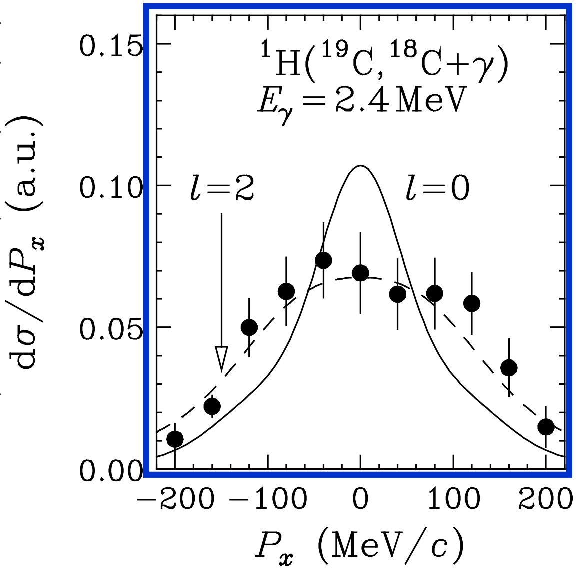
Neutron in s-wave

$^{17}C(E_x=0.21MeV)$:$1/2^+$



Neutron in d-wave

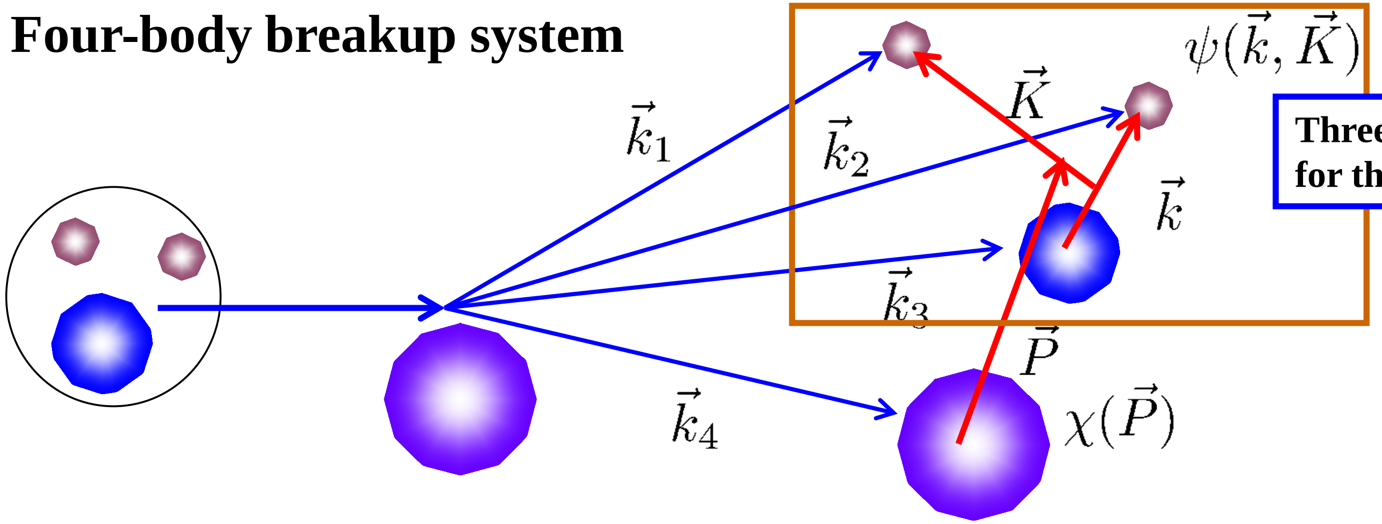
$^{18}C(E_x=4MeV)$:$2^+,3^+$



Y. Kondo, T. Nakamura, Y. Sato, T. M. N. Aoi et al., Phys. Rev. C79, 014602 (2009)

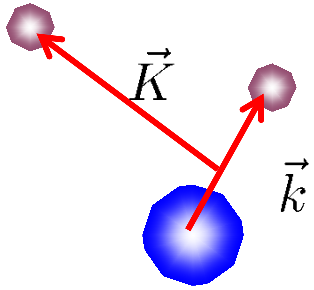
Smoothing Function Method (4-body)

Four-body breakup system



Final state

Three-body breakup continuum for the projectile



$\psi(\vec{k},\vec{K})$

Continuous T-matrix elements

$$T(\vec{k},\vec{K},\vec{P}) = \langle \psi(\vec{k},\vec{K})\chi(\vec{P})|U|\Psi^{\mathrm{CDCC}}\rangle \approx \sum_{\gamma} f_{\gamma}(\vec{k},\vec{K})T_{\gamma}$$

Wave functions of three-body breakup continuum are needed.

Smoothing Factor

$$f_{\gamma}(\vec{k},\vec{K}) = \langle \psi(\vec{k},\vec{K})|\phi_{\gamma}\rangle$$

Complex-Scaling Method

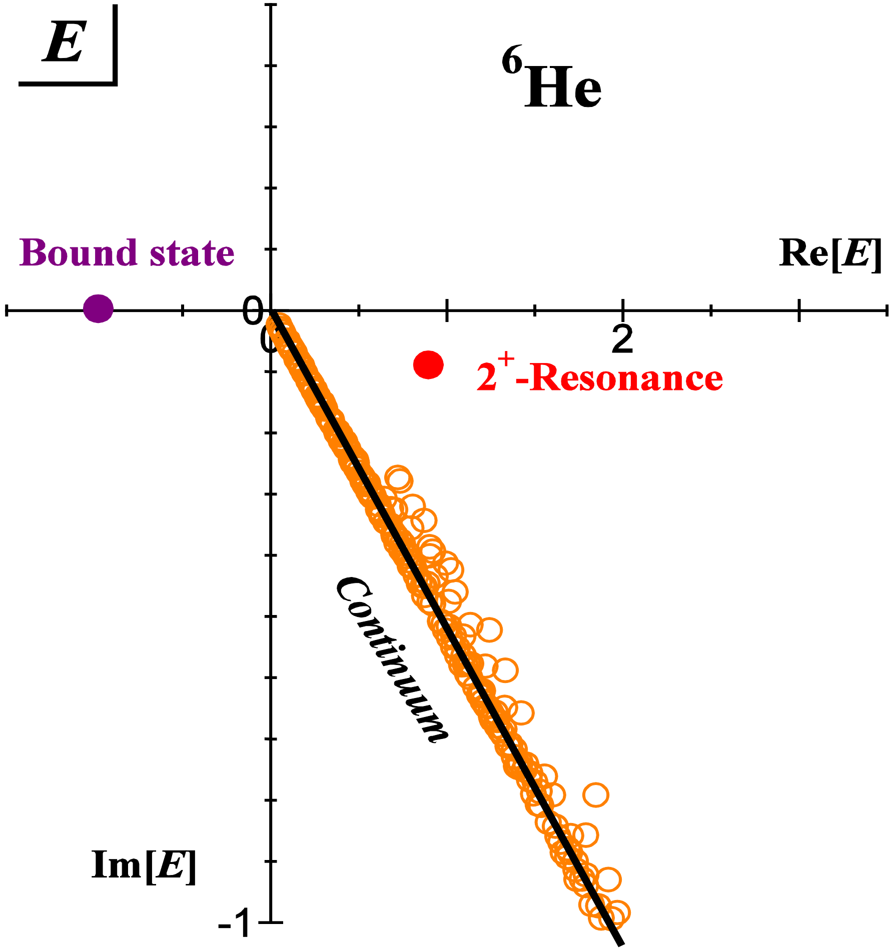
Complex-scaling operator: U^θ

$$U^\theta f(r) = e^{i3/2\theta} f(re^{i\theta})$$

Coordinate: $r \rightarrow re^{i\theta}$

Momentum: $k \rightarrow ke^{-i\theta}$

Useful for searching many-body resonances



Green's function with Complex-Scaling Method (CDCS Green's function)

$$\mathcal{G}^{(-)}(E,\xi,\xi') = \frac{1}{E-H-i\epsilon} \approx \sum_\nu U^{-\theta} \frac{|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|}{E-E_\nu^\theta} U^\theta$$

New Smoothing Procedure with *CSM*

$$\frac{d\sigma}{dE} = \int T^{\dagger}(E')T(E')\delta(E-E')dE' = \frac{1}{\pi}\mathrm{Im}\mathcal{R}(E)$$

$$T(E) = \langle\psi^{(-)}(E,\xi)\chi_C^{(-)}(\mathbf{R})|V|\Psi^{(+)}(\xi,\mathbf{R})\rangle$$

Final state of the projectile

Response function

$$\mathcal{R}(E) = \int d\xi d\xi' \langle\Psi^{(+)}(\xi,\mathbf{R})|V^*|\chi_C^{(-)}(\mathbf{R})\rangle_{\mathbf{R}}\mathcal{G}^{(-)}(E,\xi,\xi')\langle\chi_C^{(-)}(\mathbf{R})|V|\Psi^{(+)}(\xi,\mathbf{R})\rangle_{\mathbf{R}}$$

Green's function with Complex-Scaling Method (CDCS Green's function)

$$\mathcal{G}^{(-)}(E,\xi,\xi') = U^{-\theta}\frac{1}{E-H^{\theta}-i\epsilon}U^{\theta} \approx \sum_{\nu} U^{-\theta}\frac{|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|}{E-E_\nu^\theta}U^{\theta}$$

$$\mathcal{G}^{(-)}(E,\xi,\xi') \approx \sum_{\nu}\sum_{i,j}|\Phi_i\rangle\frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^{\theta}|\Phi_j\rangle}{E-E_\nu^\theta}\langle\Phi_j|$$

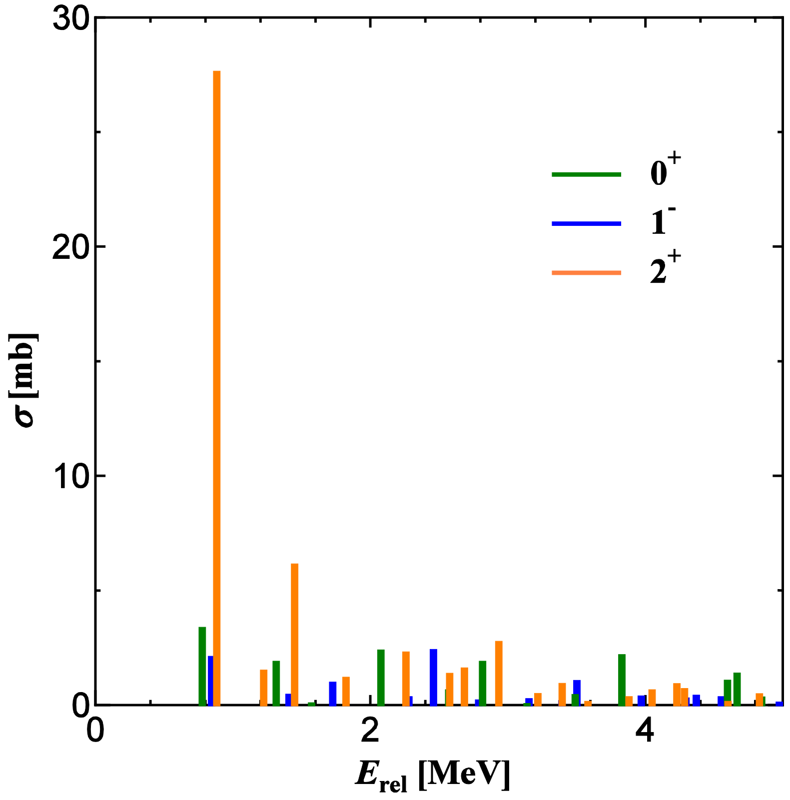
$$\mathcal{R}(E) = \sum_{\nu}\sum_{i,j}\langle\Psi^{(+)}|V^*|\chi_C^{(-)}\Phi_i\rangle\frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^{\theta}|\Phi_j\rangle}{E-E_\nu^\theta}\langle\Phi_j\chi_C^{(-)}|V|\Psi^{(+)}\rangle$$

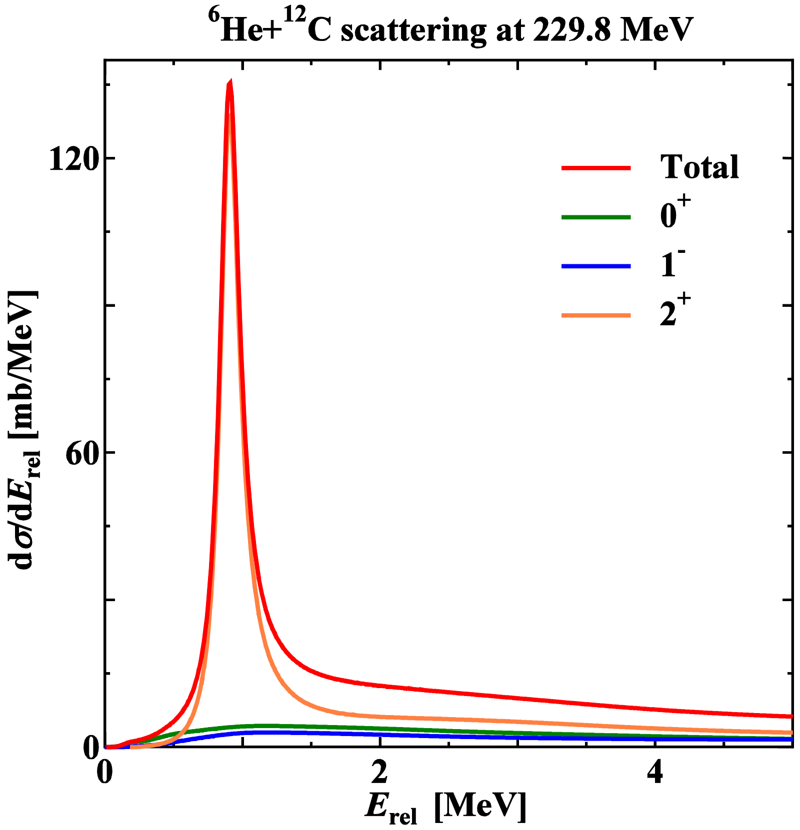
T-matrix calculated by CDCC

Differential Breakup Cross Section

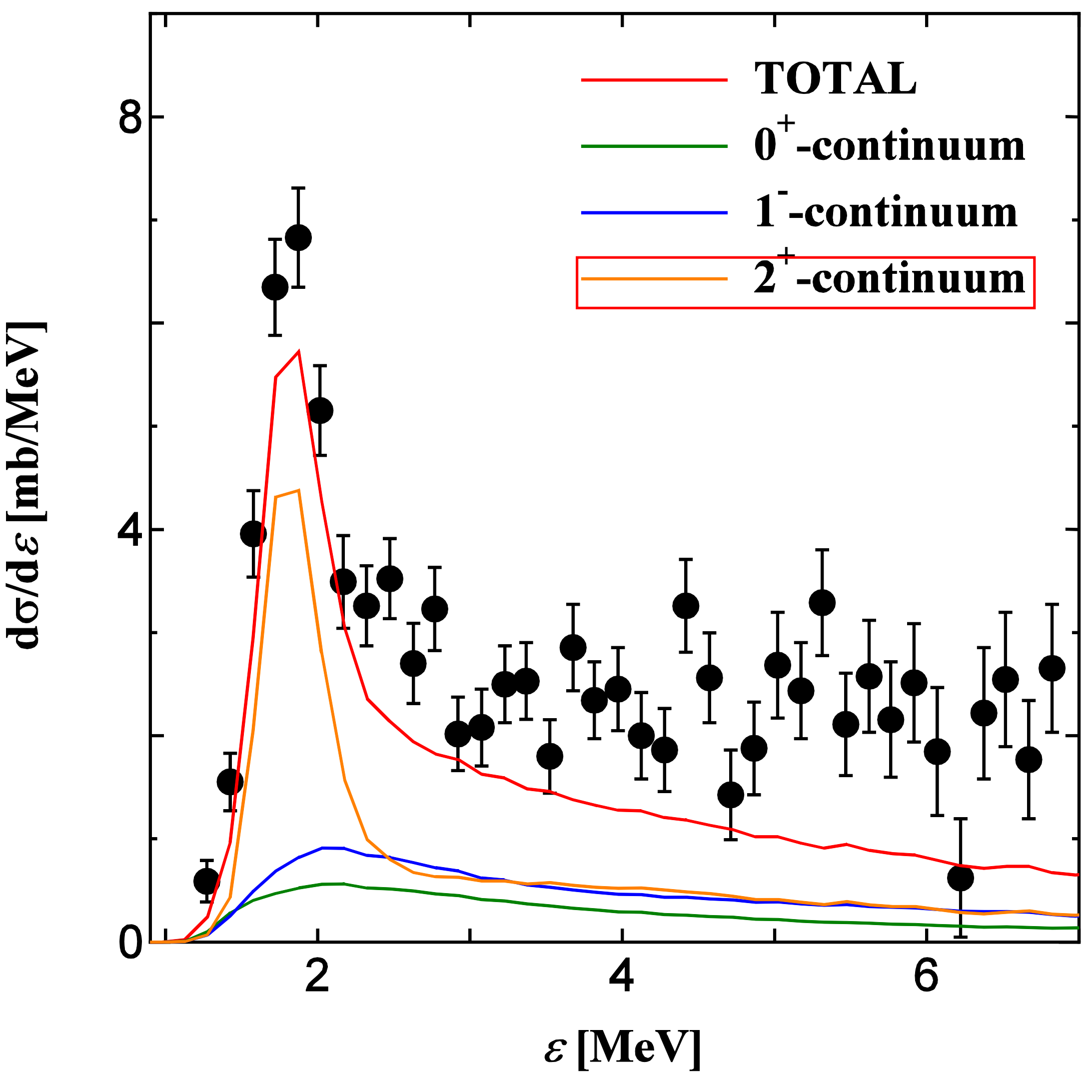
New description of differential breakup cross section

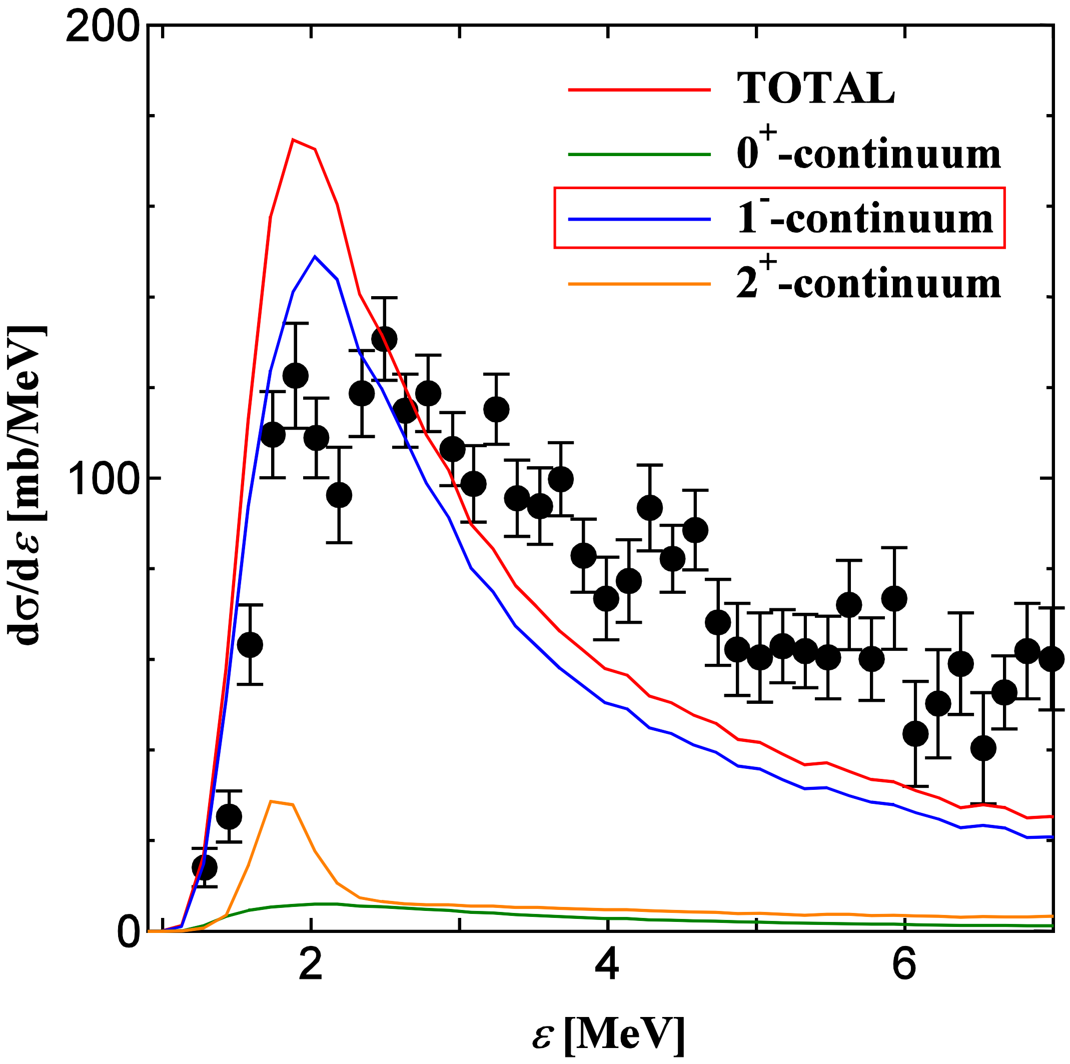
$$\frac{d\sigma}{dE} = \frac{1}{\pi}\mathrm{Im}\sum_{\nu}\sum_{i,j} T_i^{\mathrm{CDCC}\dagger}\frac{\langle\Phi_i|U^{-\theta}|\Phi_\nu^\theta\rangle\langle\tilde{\Phi}_\nu^\theta|U^\theta|\Phi_j\rangle}{E-E_\nu^\theta}T_j^{\mathrm{CDCC}}$$





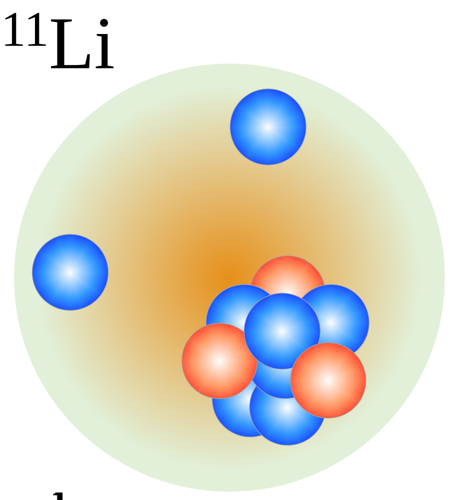
Breakup Cross Section for $^{6}He+^{12}C$, ^{208}Pb





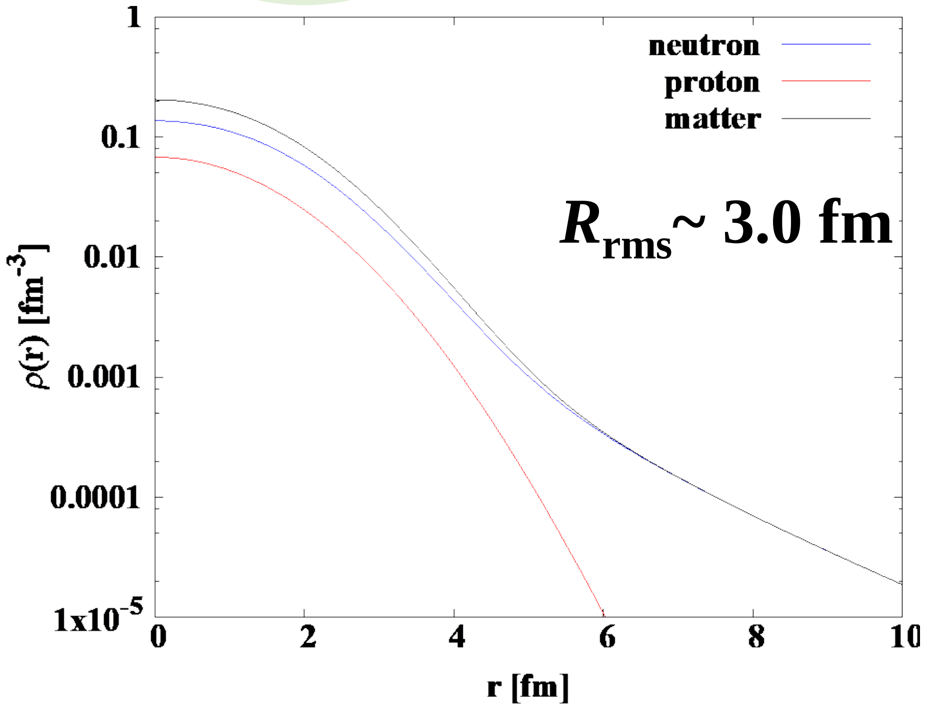
Exp. data from PRC59, 1252 (1999), T. Aumann *et al.*

Three-body model of ^{11}Li

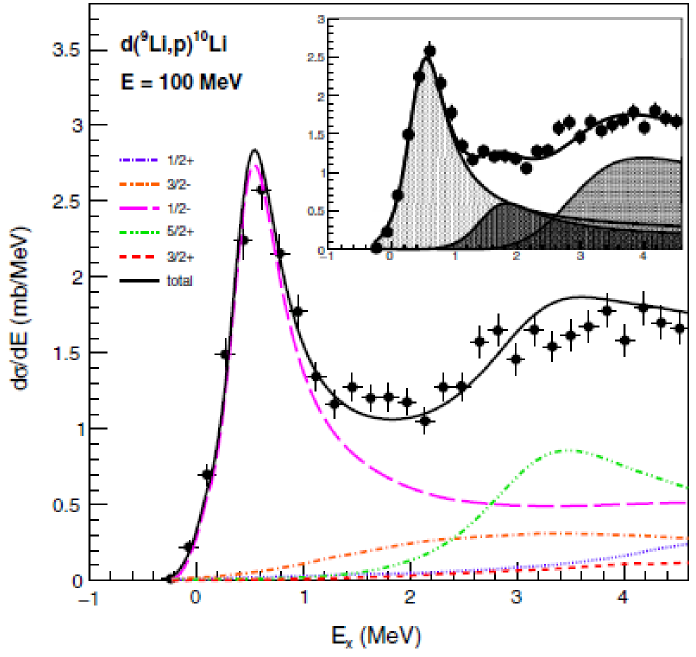


^{11}Li : $n + n + {}^{9}$Li (spinless)

V_{NN} : Minnesota interaction
V_{NC} : Esbensen and Bertsch, NPA 542, 310
V_{NNC}: to optimize $S_{2n} \sim 0.37$ MeV



^{10}Li resonance ~ 0.5 MeV



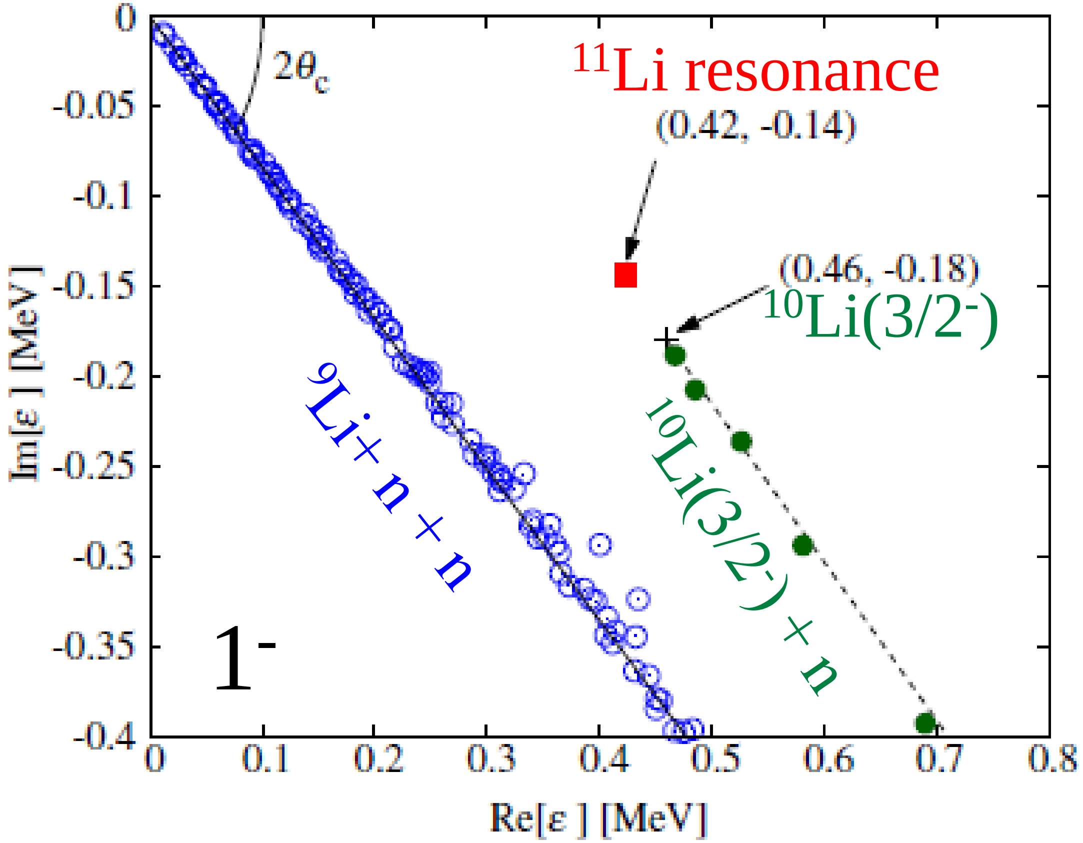
M.Cavallaro et al., PRL 118,012701 (2017)

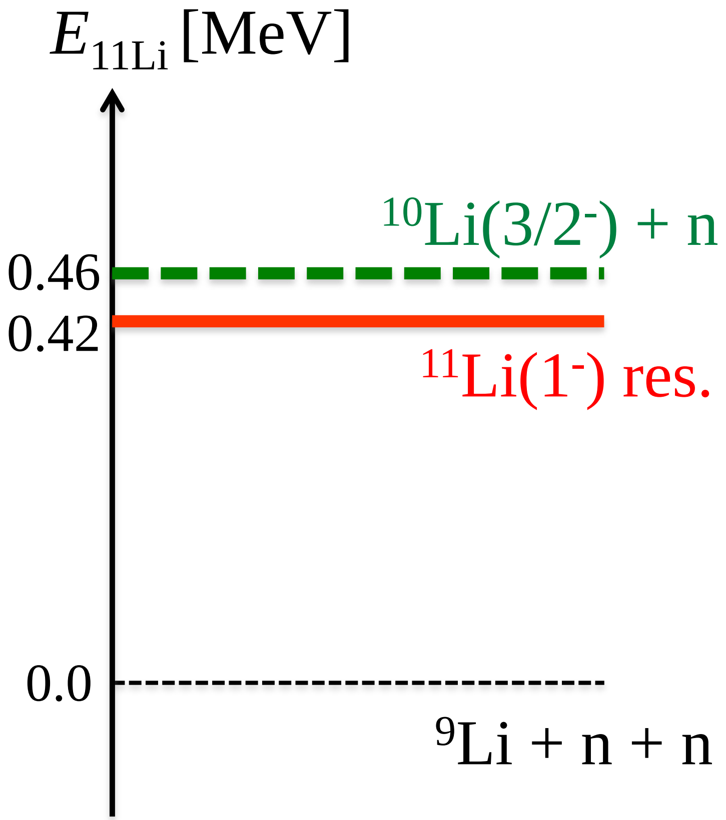
Resonance of ^{11}Li

- Complex-scaling method:

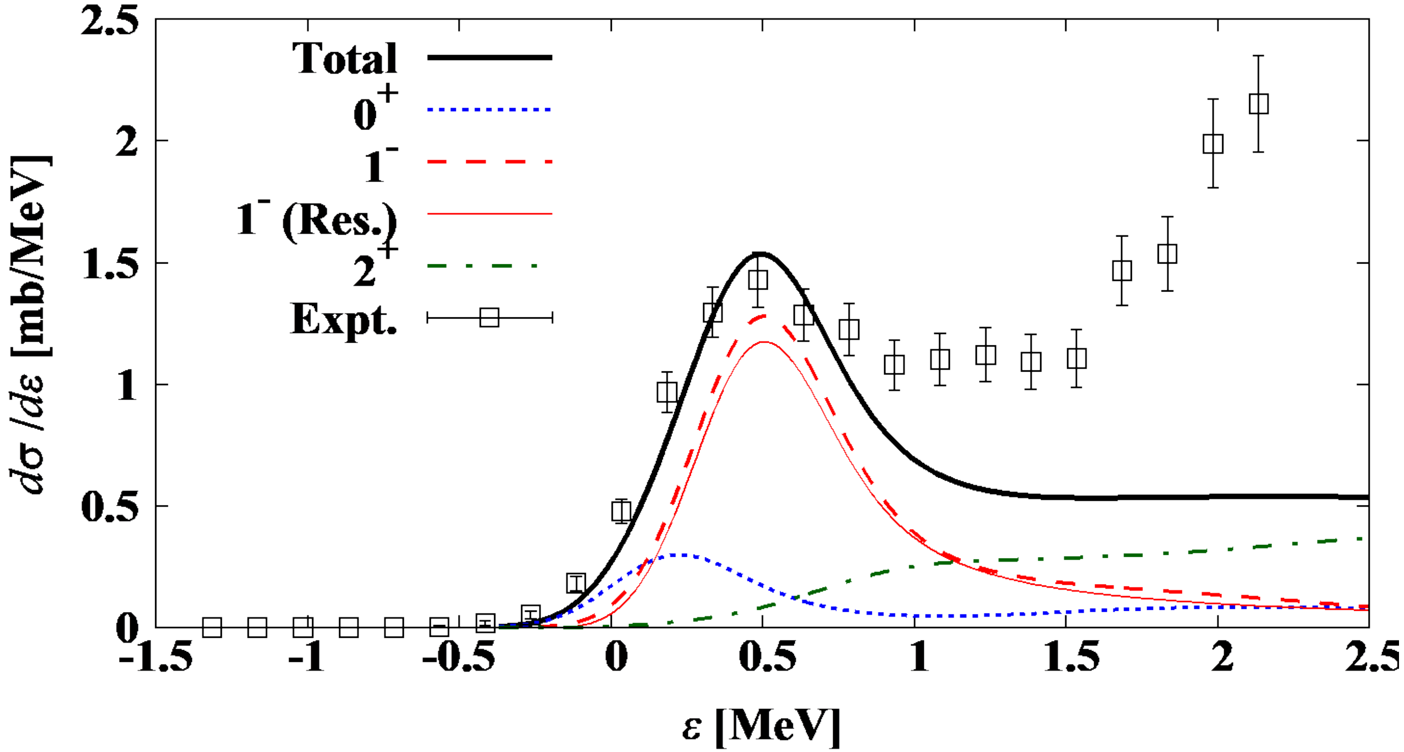
  Aoyama, Myo, Kato, and Ikeda, Prog. Theor. Phys. 116, 1 (2006)

- ^{11}Li resonance: S. N. Ershov et al., Phys. Rev. C 70, 054608 (2004).
  E. C. Pinilla et al., Phys. Rev. C 85, 054610 (2012).





Energy spectrum of ^{11}Li(p, p’)

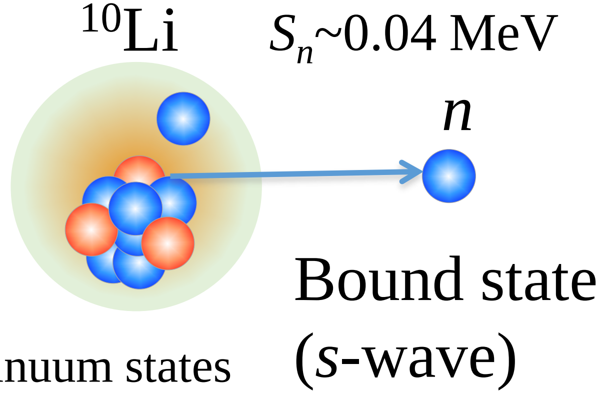


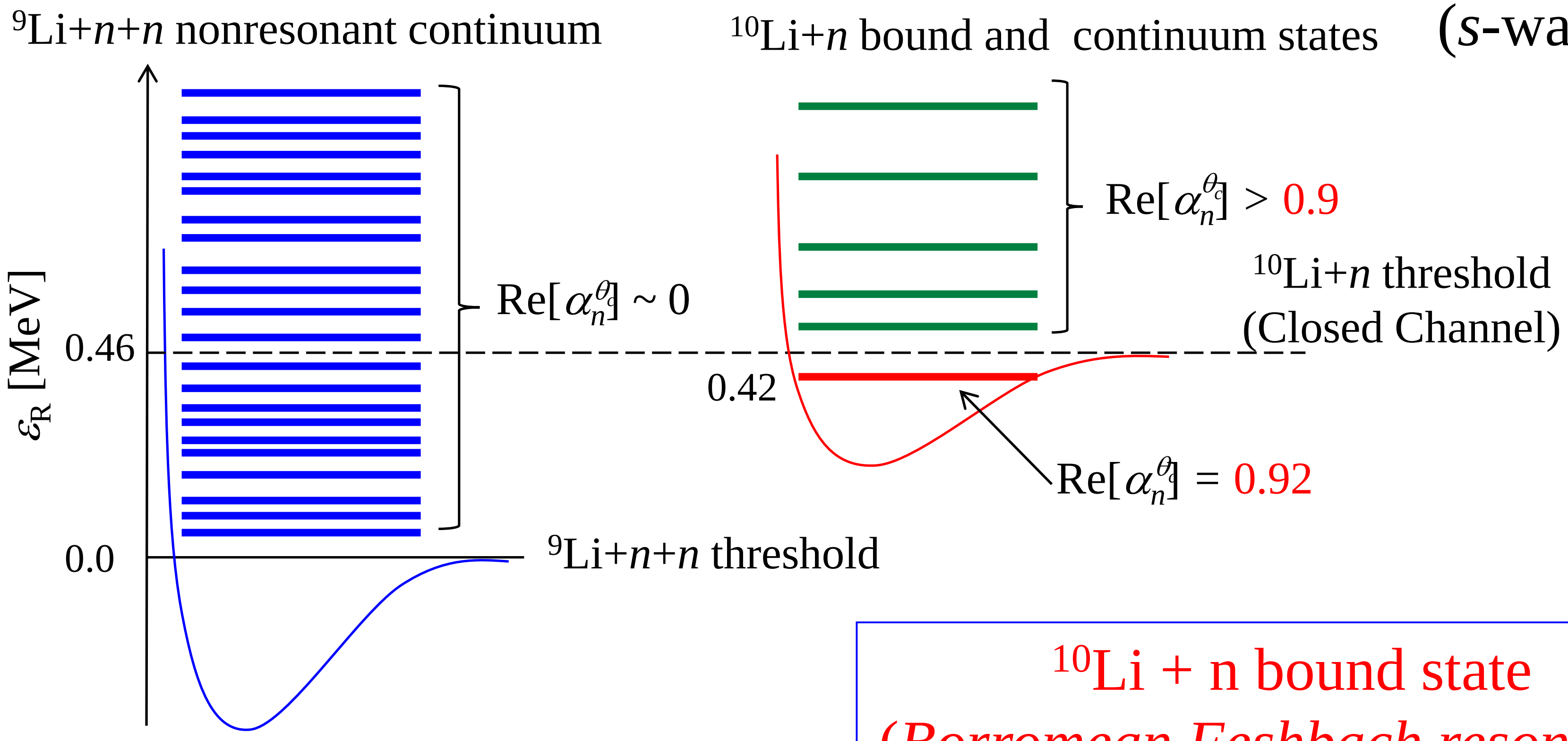
- The contribution of the dipole resonance dominates the low-lying peak.
- The width of the low-lying peak is reproduced by taking into account non-resonant components.

Borromean Feshbach Resonance

$$\psi_{res}(^{11}Li) = \left[\phi_{res}(^{10}Li) \otimes \chi_n(\ell = 0)\right]$$

0.42 MeV   0.46 MeV   -0.04 MeV





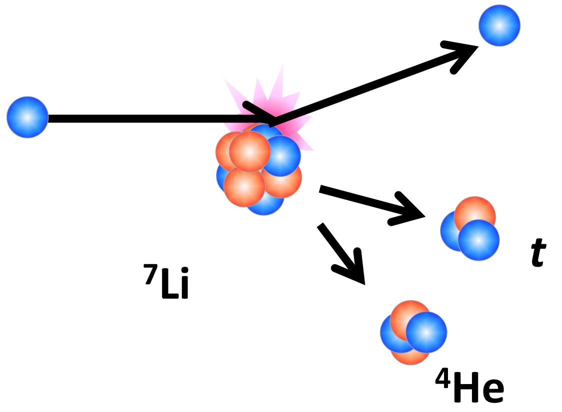
^{10}Li + n bound state

(*Borromean Feshbach resonance*)

Application to Nuclear Engineering

❑ *Background*

- ***Lithium** is an important element relevant to not only a tritium breeding material in **DT fusion reactors** but also a candidate for target material in the intense neutron source of **IFMIF**.*
- ***Accurate nuclear data of nucleon induced reactions on ^{6,7}Li are currently required for incident energies up to 150 MeV.***
- ***Statistical model is not applicable to Li scattering because the clustering structure is important for Li***



❑ *CDCC calculation*

- ^{6,7}Li structure

  ^{6}Li = ^{4}He + *d*

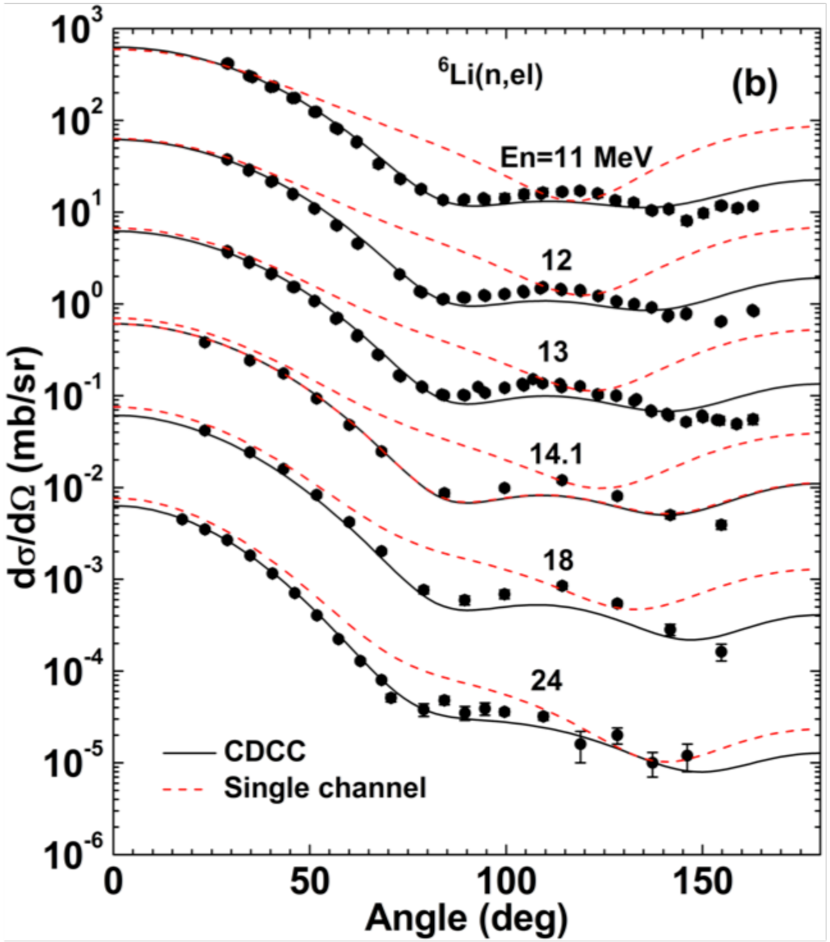
  ^{7}Li = ^{4}He + *t* ***2-body cluster model***

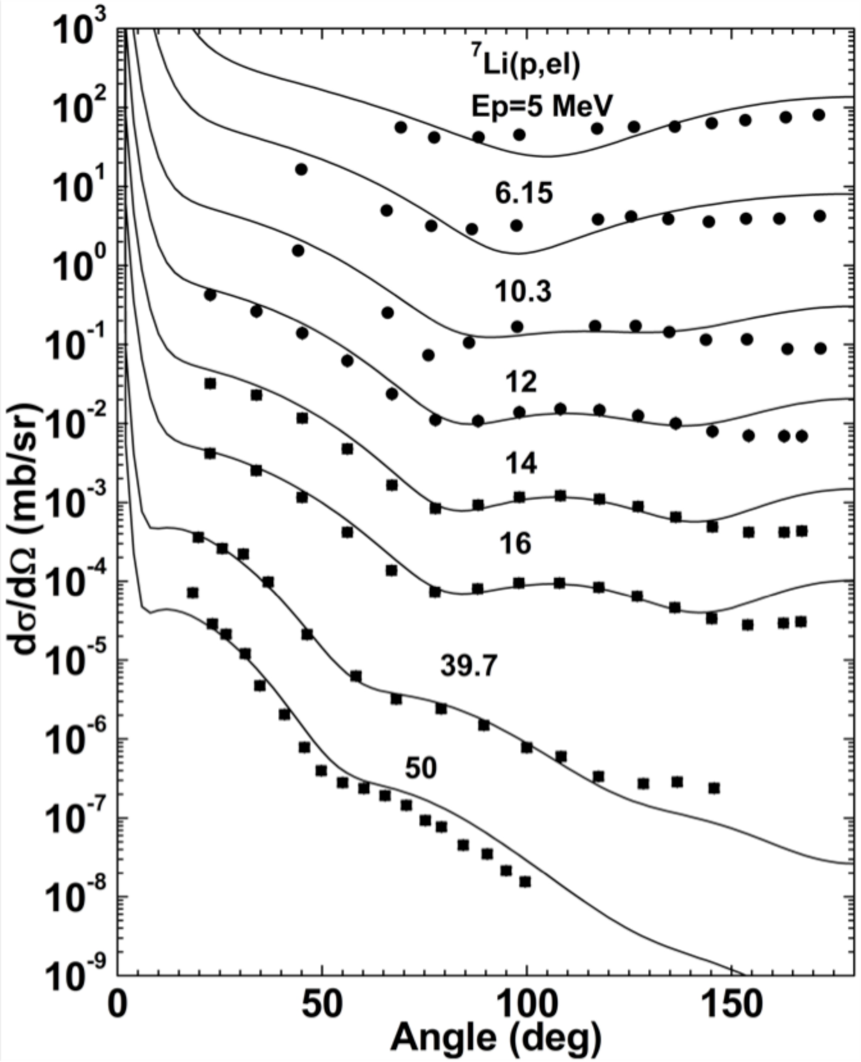
- Nucleon-^{6,7}Li interaction

  Single folding model with ***JLM***

This work has been performed at Hokkaido Univ (JCPRG) and Kyushu Univ..

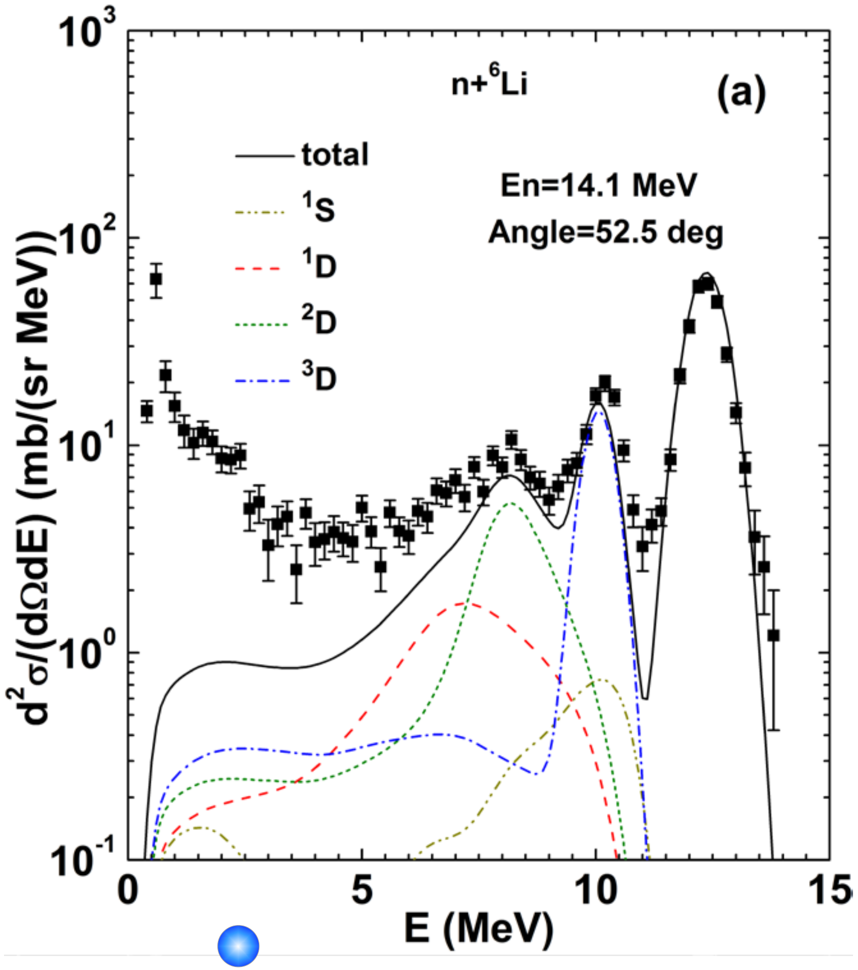
Elastic Scattering of nucleon from $^{6,7}Li$

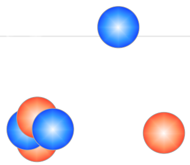




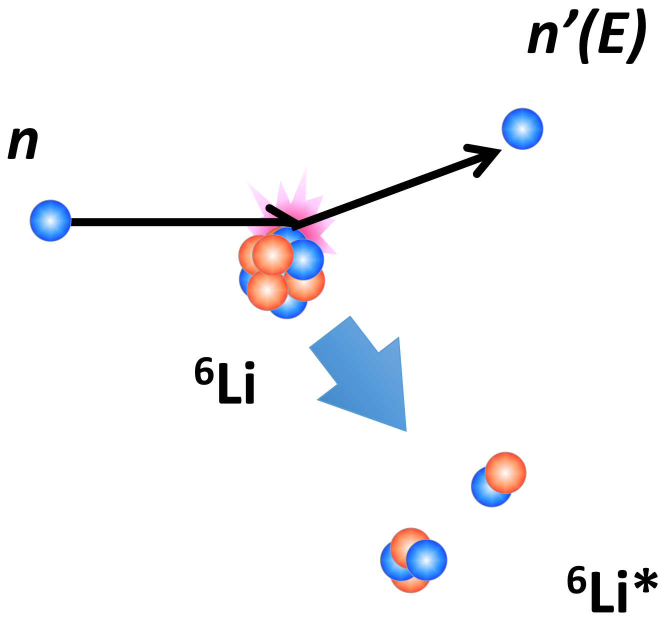
Guo, Watanabe, Matsumoto, Ogata, Yahiro
Phys. Rev. C87, 024610 (2013)

Cross Section for Neutron Emission





^{6}Li 4-body CDCC analysis



Neutron emitted spectrum represents the excited structure of ^{6}Li, and CDCC calculation well reproduce the data.

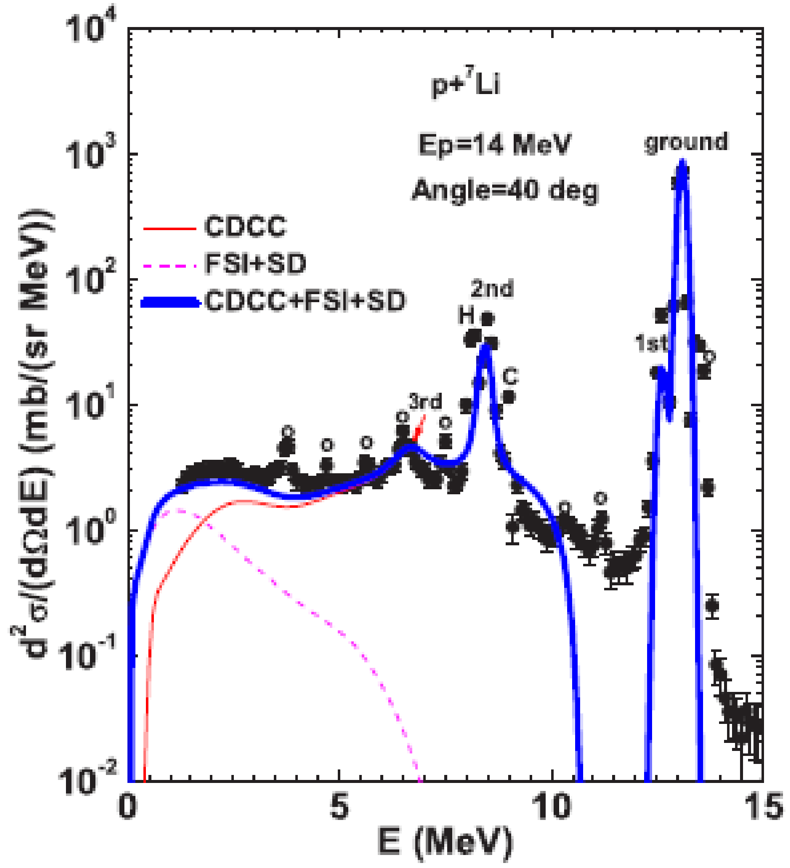
Matsumoto, Ichinkhorloo, Hirabayashi, Chiba, Kato
Phys. Rev. C83, 064611 (2011)
Ichinkhorloo, Hirabayashi, Kato, Aikawa, Matsumoto
Phys. Rev. C86, 064604 (2012).

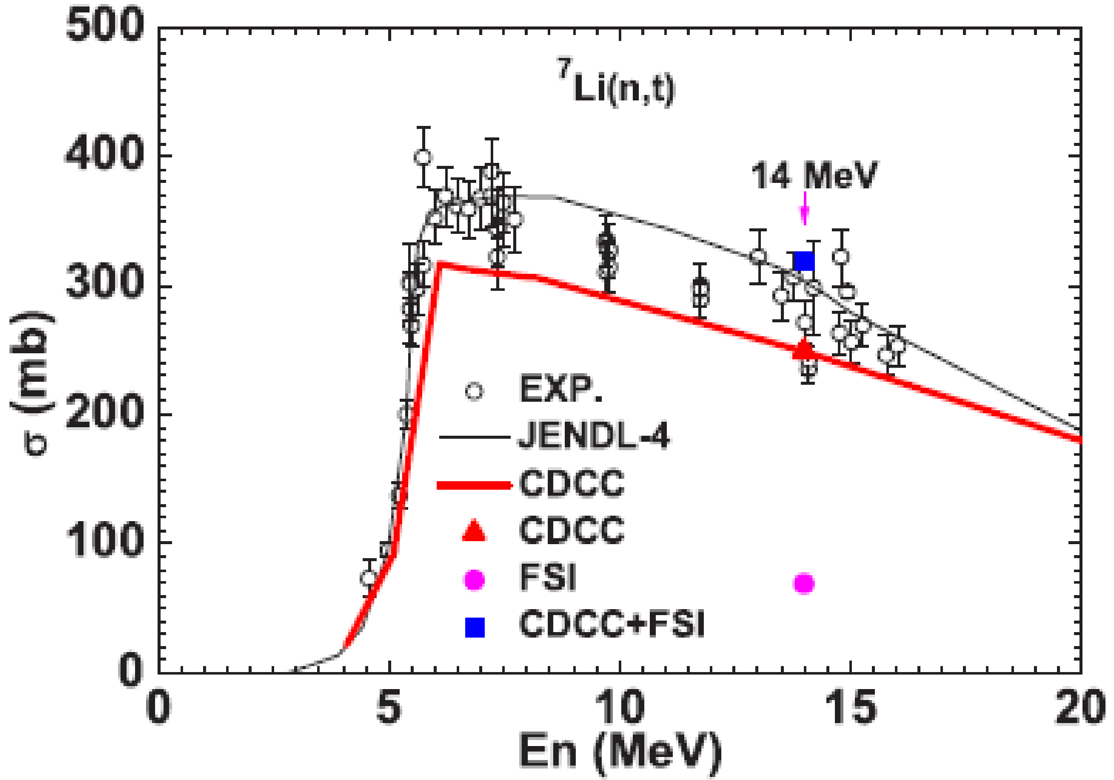
CDCC + FSI model + SD model

CDCC+ Final-state interaction (FSI) model
Sequential decay (SD) model

$$p(n) + {}^7\mathrm{Li} \rightarrow p'(n') + {}^7\mathrm{Li}^* \rightarrow p'(n') + \alpha + t, \qquad (a)$$

$$p(n) + {}^7\mathrm{Li} \rightarrow t + {}^5\mathrm{Li}({}^5\mathrm{He}) \rightarrow t + p'(n') + \alpha, \qquad (b)$$





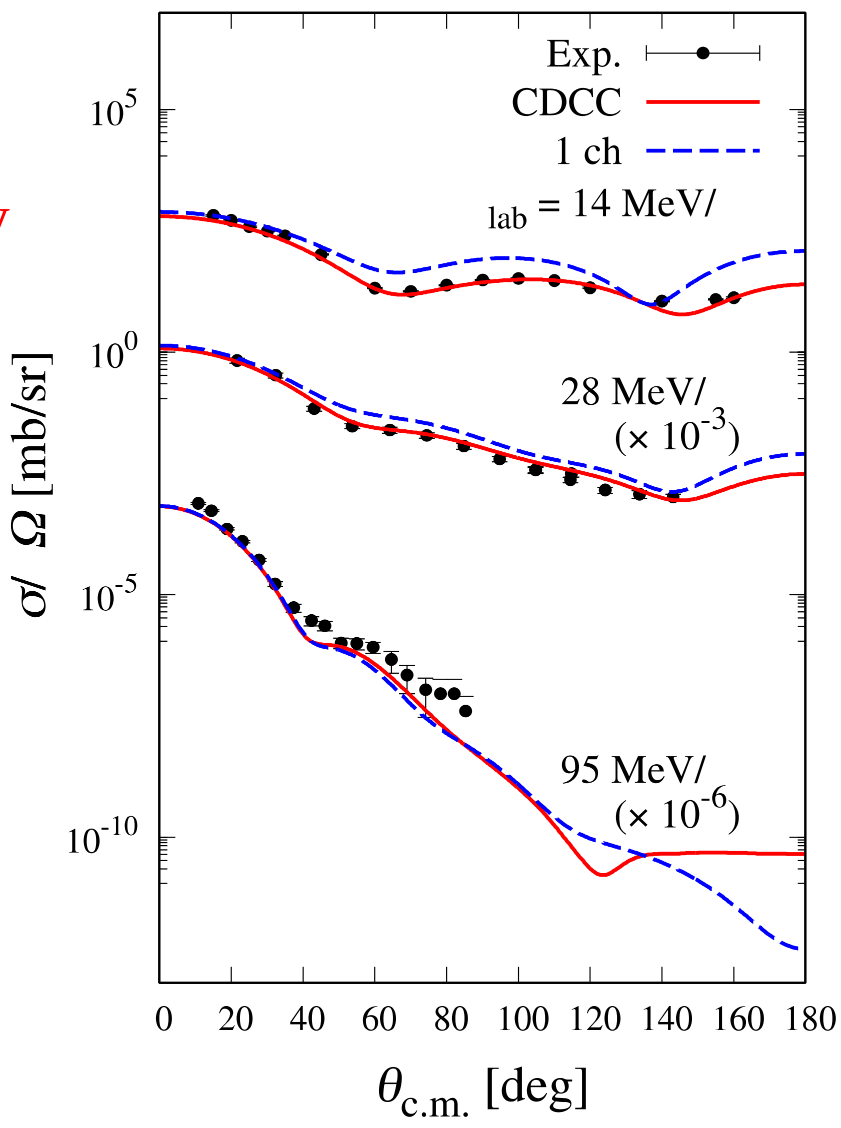
Guo, Watanabe, Matsumoto, Nagaoka, Ogata, Yahiro
Phys. Rev. C99, 024610 (2019)

Elastic scattering of nucleon from ^{12}C

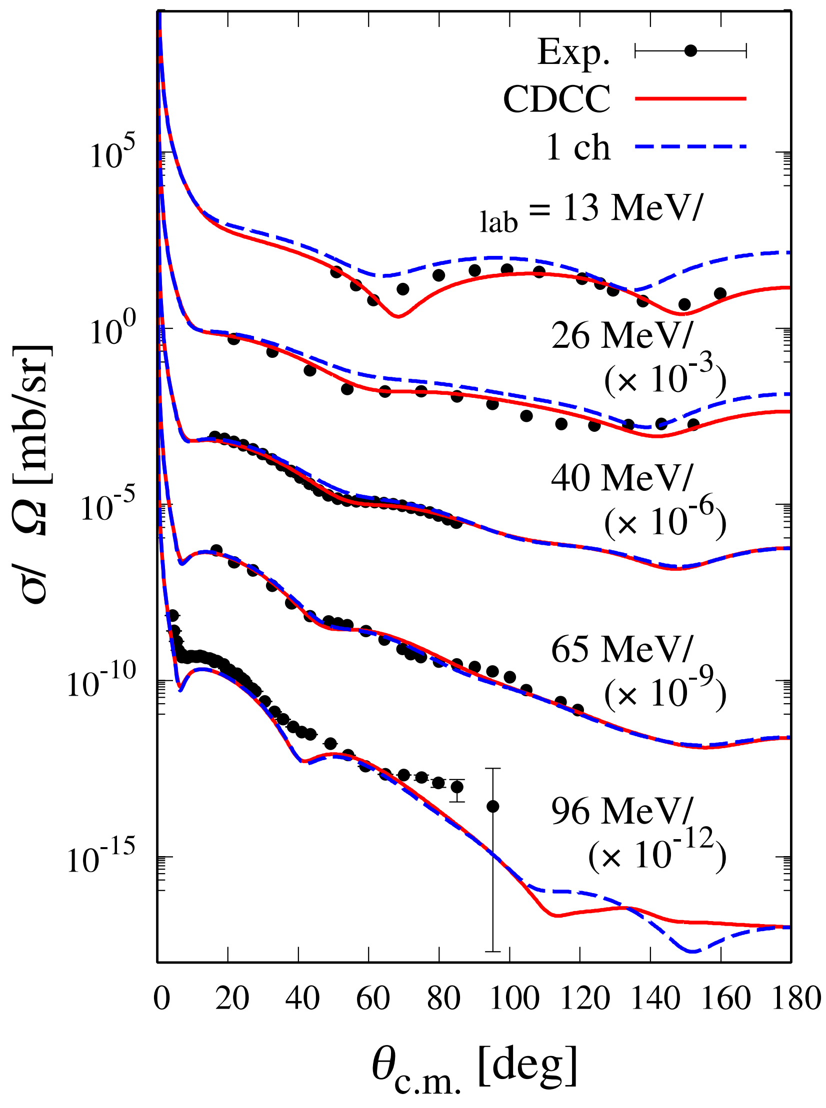
^{12}C=3α

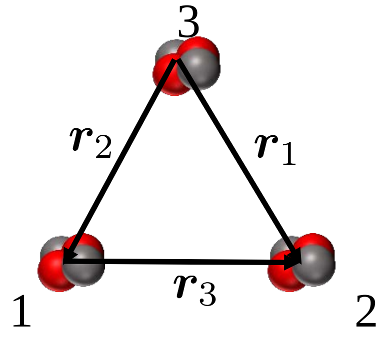
Four-body

n-^{12}C

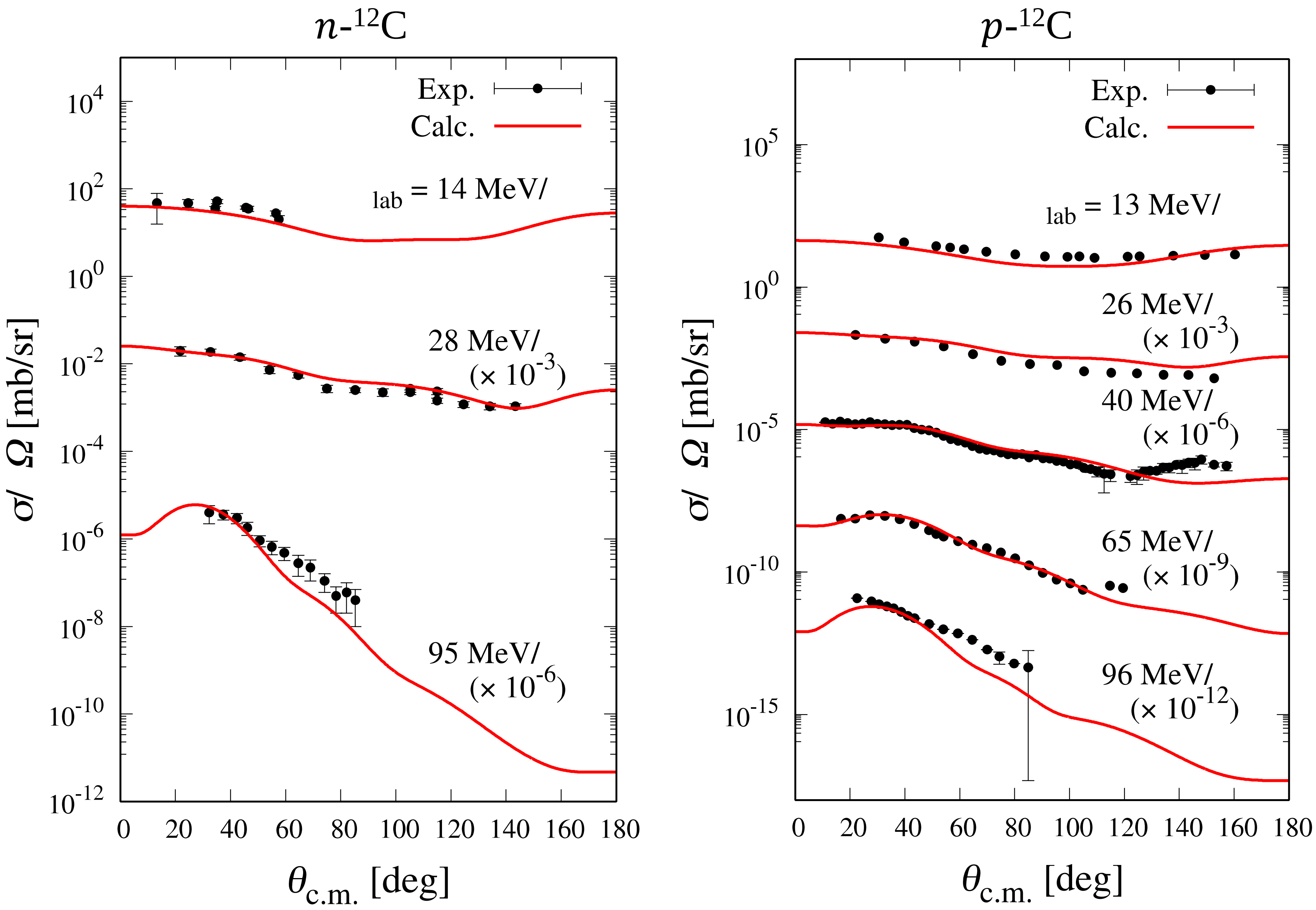


p-^{12}C

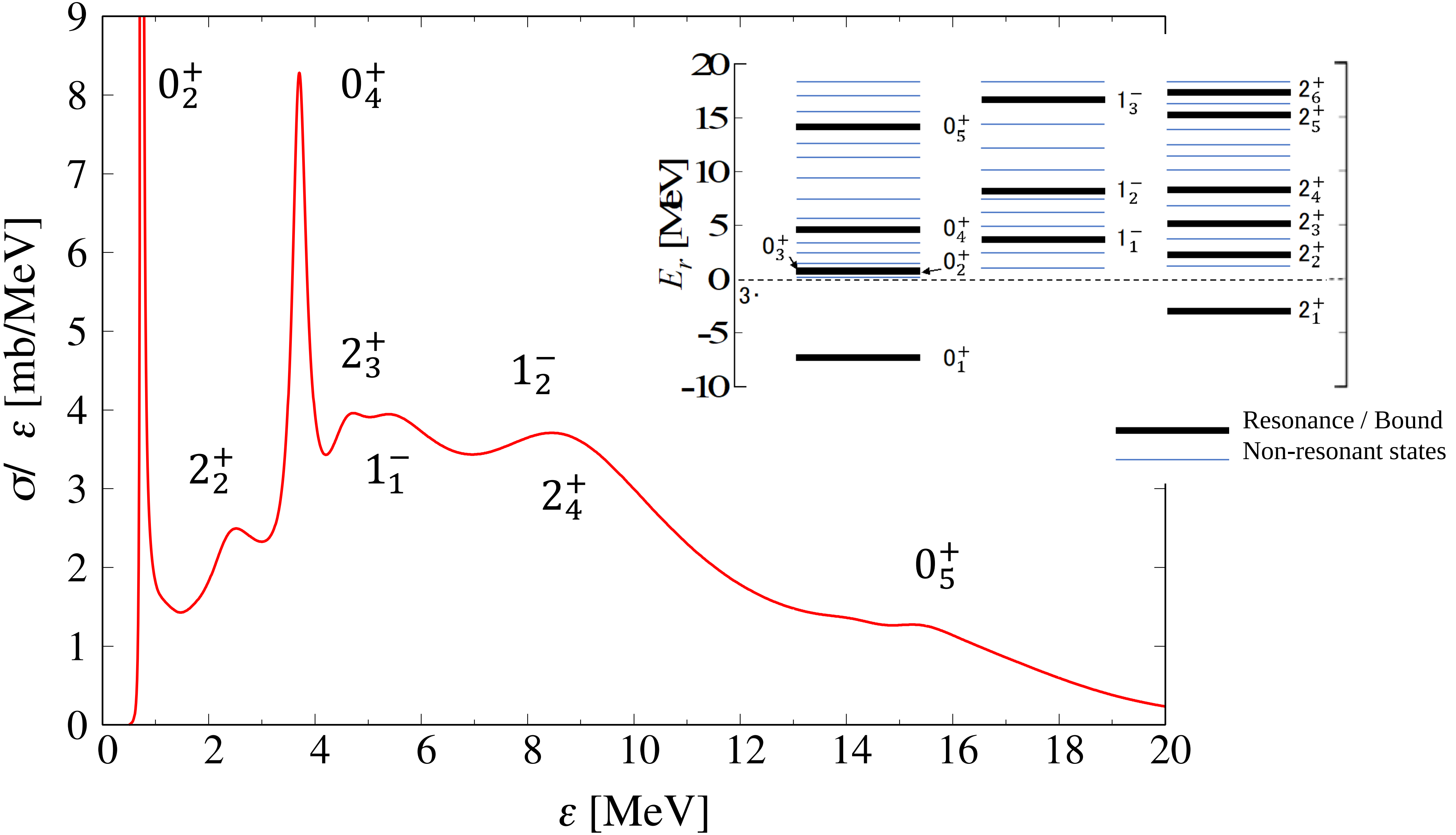




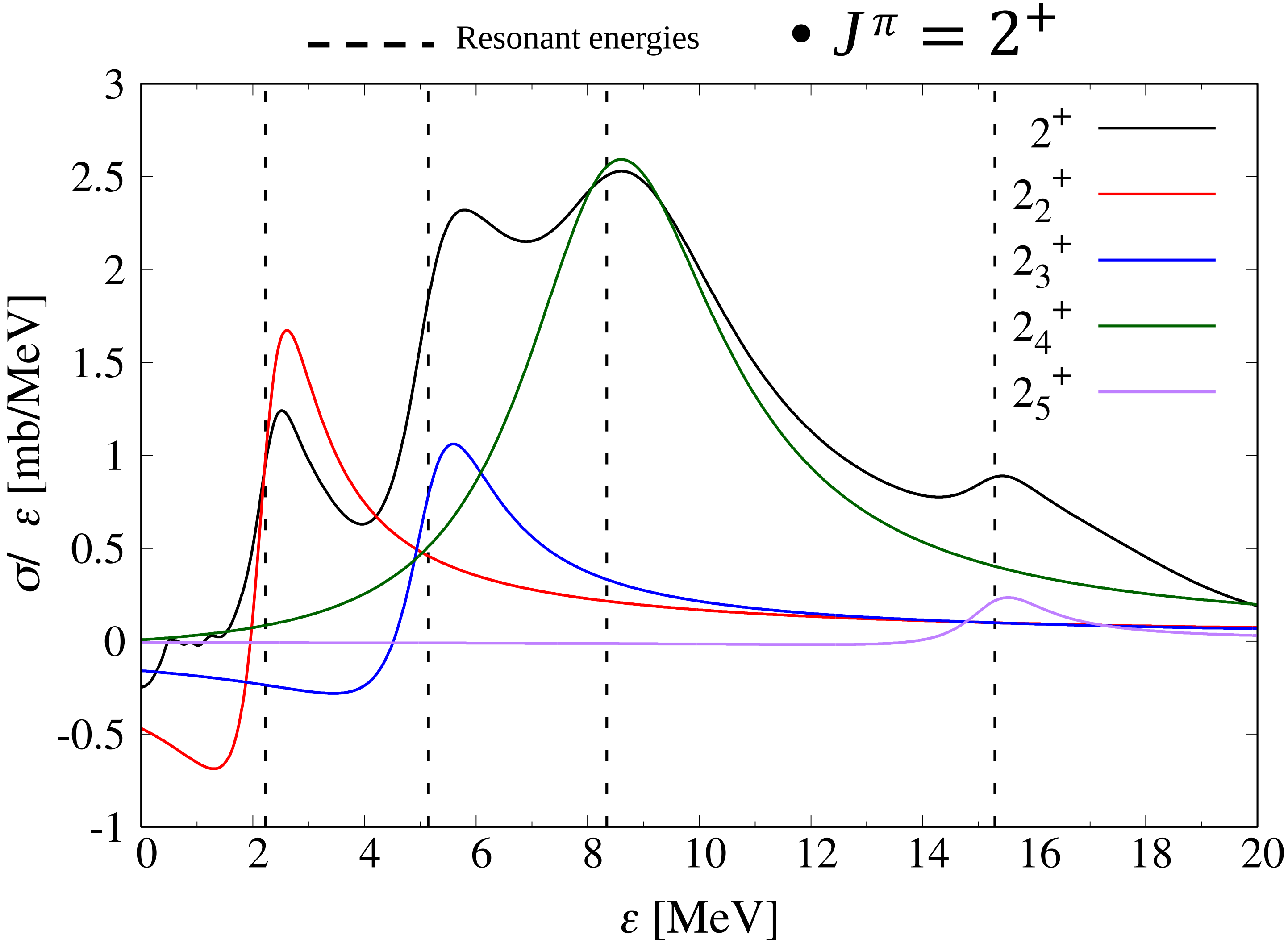
Inelastic scattering to 2^+ for nucleon+^{12}C

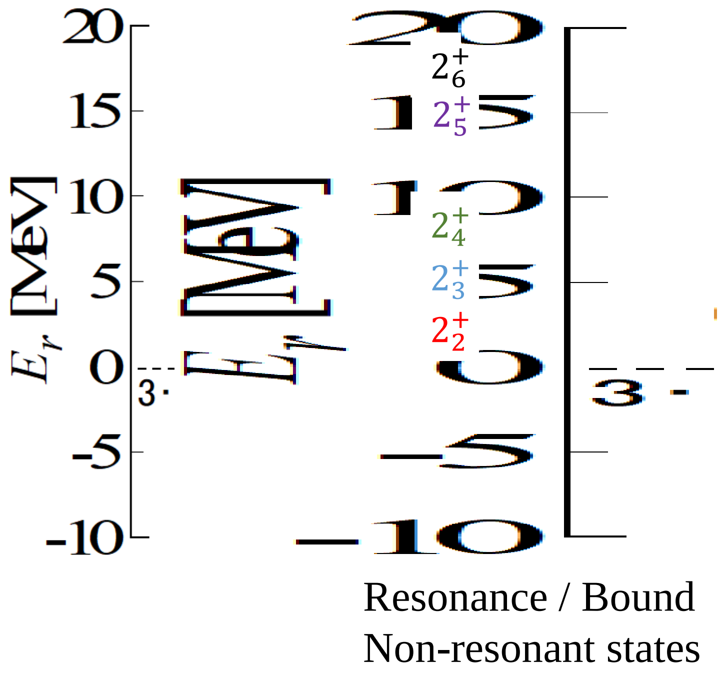


Energy Spectrum for n+^{12}C @ 28 MeV



Breakup Cross Section to 2+ continuum





✓ The peak positions of the cross section are higher than the corresponding resonant energies about 200-500 keV.

Summary

- CDCC is one of reliable method for treating breakup process accurately for many-body breakup reactions.

- CDCC is useful for not only study on unstable nuclei but also estimation of nuclear data.

  - ✓ Momentum distribution (^{17}C, ^{18}C) Three-body
  - ✓ Energy spectrum (^{6}He, ^{11}Li) Four-body
  - ✓ Nucleon-$^{7,6}Li$ Three-body
  - ✓ Nucleon-^{12}C Four-body

→ Resonance & Energy Spectrum